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An interpretation of the stomach contents of fish in relation to prey abundance

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Abstract

Simple hypotheses on digestion together with the food preference function stated in Andersen & Ursin 1977 are used for an interpretation of the stomach contents of fish. The derived formulas are illustrated by means of five examples.

Hypotheses

Any interpretation of the stomach contents of fish has to be in accordance with the knowledge on consumption and digestion.

In the ideal situation with consumption and digestion completely known only one interpretation is possible. As our knowledge is very limited many interpretations are possible using different sets of hypothesis on consumption and digestion. One set of hypotheses in agreement with the consumption model formulated in Andersen & Ursin 1977 is:

1. *Consumption*

The consumption rate for a single size class w_{pr} of a prey pr as consumed by a predator Pr weighing W_{Pr} is:

$$\text{Consumption rate } (pr, Pr, w_{pr}, W_{Pr}) = \frac{\rho(pr, Pr, W_{Pr})\exp(-(\log(W_{Pr}/w_{pr}) - \eta(pr, Pr, W_{Pr}))^2/2/\sigma(pr, Pr, W_{Pr})^2)}{\phi(Pr, W_{Pr}) + Q(Pr, W_{Pr})} * N(pr, w_{pr})\, w_{pr} h(Pr)\, W_{Pr}^{m(Pr)} \tag{i}$$

as found from the formulas (4), (15), (16), and (18) in Andersen & Ursin 1977.

Here $prfu = \rho\exp(-(\log(W/w) - \eta)^2/2/\sigma^2)$ is Pr's preference function as regards the prey species pr. ρ, η and σ^2 are parameters depending on predator and prey species and (perhaps) on the size of the predator. ϕ is the 'available' food for the predator and

$$\phi(Pr, W_{Pr}) = \sum_{\substack{pr\, \varepsilon \text{ all prey species} \\ \text{and sizes}}} prfu(pr, w_{pr}, Pr, W_{Pr})\, N(pr, w_{pr})\, w_{pr}\,. \tag{ii}$$

$N(pr,w_{pr})$ is the number of prey species pr weighing w_{pr}. Q is the half saturation function from the feeding level equation for (Pr,W_{Pr}):

feeding level $= f = \phi/(\phi + Q)$.

(h,m) are the coefficient and exponent of ingestion for (Pr,W_{Pr}).

The consumption is determined by (h,m), the set of ρ's, the set of (η,σ^2)'s, the set of N's, and the set of Q's. The ρ, N and Q sets have however only to be known to two constants:

if

$$\{(\rho),(N),(Q)\}$$

is one set then

$$\{(c_1\rho),(c_2N),(c_1c_2Q)\}$$

will do just as well.

2. *Digestion*

The digestion rate for a food item is the present weight of the food item multiplied by a species parameter κ:

digestion rate for a food item $= \kappa(pr,Pr,W_{Pr}) *$ (present weight of food item).

3. *Stomach contents*

The weight of the stomach contents, $B_{st}(Pr,W_{Pr},pr,w_{pr},t)$, of prey (pr,w_{pr}) for a predator (Pr,W_{Pr}) satisfies the differential equation:

$$\mathrm{d}B_{st}/\mathrm{d}t = \text{consumption} - \kappa B_{st}$$

for all (pr,w_{pr}), where w_{pr} is the weight at ingestion.

4. *Stomach data*

The size of B_{st} can be found by averaging the stomach contents of a large number of predators of the same species and size. The change in the average stomach content is very slow or approximately

$$\mathrm{d}B_{st}/\mathrm{d}t = 0$$

for all prey weights and species.

The digestion is so speedy that all N's, W's and w's can be taken as constants for the time interval a prey individual can be traced in the stomach of a predator. This is the reason why the N's do not appear and t is taken as an explicit argument in B_{st}.

As B_{st} is independent of t in the equilibrium we now find:

$$B_{st}(Pr,W_{Pr},pr,w_{pr})\,\kappa(pr,Pr,W_{Pr}) = \frac{\rho(pr,Pr,W_{Pr})\exp(-(\log(W_{Pr}/w_{pr}) - \eta(pr,Pr,W_{Pr}))^2/2/\sigma(pr,Pr,W_{Pr})^2)}{\phi(Pr,W_{Pr}) + Q(Pr,W_{Pr})}$$
$$* \; N(pr,w_{pr})\,w_{pr}\,h(Pr)\,W_{Pr}^{m(Pr)} \qquad \text{(iii)}$$

or

$$\rho \exp(-(\log(W/w) - \eta)^2/2/\sigma^2) = prfu = \kappa B_{st} * (\phi + Q)/N/w/h/W^m . \quad \text{(iv)}$$

This means that *prfu* is proportional to the ratio between stomach weight and prey biomasses for each ingestion prey size and further that when κ is independent of prey species:

$$\kappa(\text{weight of total stomach contents}) = \text{consumption} = fhW^m \quad \text{(v)}$$

or

$$\kappa(\phi + Q) = \phi h W^m/(\text{weight of total stomach contents}). \quad \text{(vi)}$$

It has so far been taken for granted that the digestion takes place in the stomach only and that the food stays in the stomach until it is completely digested. It has also been assumed that the identification[1] of the stomach contents is possible independent of the time a food item has been due to digestion. To relax these unrealistic assumptions we proceed as follows: Let the fate of a food item weighing w_{pr} when consumed be as follows: In a timespan t_l no digestion takes place (digestion lags behind ingestion). The item is recognizable in a timespan t_0 where digestion takes place. The item is removed from the stomach after a timespan $t_l + t_{00}$ ($t_0 \leq t_{00}$).

t	$t + t_l$	$t + t_l + t_0$	$t + t_l + t_{00}$
ingestion	digestion starts	item unrecognizable	item leaves stomach

This makes it natural to split B_{st} up in three parts

$$B_{st} = B_u + B_{dr} + B_{dur}$$

where

B_u = undigested part
B_{dr} = digested but recognizable part
B_{dur} = digested and unrecognizable part fulfilling the differential equations

$$dB_u/dt = \text{consumption rate} - \Delta \quad (= 0 \text{ in the equilibrium})$$
$$dB_{dr}/dt = \Delta - \kappa B_{dr} - \lambda \quad (= 0 \text{ in the equilibrium})$$
$$dB_{dur}/dt = \lambda - \kappa B_{dur} - \zeta \quad (= 0 \text{ in the equilibrium})$$

where

Δ = rate items become digestable
λ = rate items become unrecognizable
ζ = rate items leave stomach.

In the equilibrium Δ, λ and ζ become constants and we find if the consumption rate is $N_0 w_{pr}$:

$$\Delta = N_0 w_{pr}$$
$$\lambda = N_0 w_{pr} - \kappa B_{dr}$$
$$\zeta = N_0 w_{pr} - \kappa(B_{dr} + B_{dur})$$
$$B_u = \int_0^{t_l} N_0 w_{pr} dt = N_0 w_{pr} t_l$$

1. A food item is identifiable or recognizable if both species and ingestion weight w_{pr} can be determined.

$$B_{dr} = \int_{t_l}^{t_l + t_0} N_0 w_{pr} \exp(-\kappa(t - t_l))\,dt = \frac{N_0 w_{pr}}{\kappa}(1 - \exp(-\kappa t_0))$$

$$B_{dur} = \int_{t_l + t_0}^{t_l + t_{00}} N_0 w_{pr} \exp(-\kappa t_0) \exp(-\kappa(t - (t_l + t_0)))\,dt$$

$$= \frac{N_0 w_{pr}}{\kappa}(\exp(-\kappa t_0) - \exp(-\kappa t_{00}))$$

as an item weighing w_{pr} at $t = 0$ has the weight $w_{pr}\exp(-\kappa t)$ to time t.

If N_u and N_{dr} are the numbers corresponding to B_u and B_{dr} we have

$$N_u = N_0 t_l$$
$$N_{dr} = N_0 t_0$$

and for the total recognizable number, $\widetilde{N}_{st}$

$$\widetilde{N}_{st} = N_u + N_{dr} = N_0(t_l + t_0).$$

We can now rewrite (iii) as

$$B_{st}\frac{\kappa}{\kappa t_l + 1 - \exp(-\kappa t_{00})} = \frac{\rho \exp(-(\log \frac{W}{w_{pr}} - \eta)^2/2/\sigma^2) N w_{pr} h W^m}{\phi + Q}$$

and in numbers

$$\widetilde{N}_{st}\frac{\kappa}{\kappa t_l + \kappa t_0} = \frac{\rho \exp(-(\log \frac{W}{w_{pr}} - \eta)^2/2/\sigma^2) N h W^m}{\phi + Q}. \qquad \text{(iii-}a\text{)}$$

When κt_l, κt_0, and κt_{00}, are constants we can rewrite (v) and (vi) as

$$\frac{\kappa * (\text{weight of recognizable part of total stomach contents})}{(\kappa t_l + 1 - \exp(-\kappa t_0))} = fhW^m \qquad \text{(v-}a\text{)}$$

$$\frac{\kappa * (\text{weight of total stomach contents})}{(\kappa t_l + 1 - \exp(-\kappa t_{00}))} = fhW^m \qquad \text{(v-}b\text{)}$$

$$\frac{\kappa * (\text{weight of recognizable part of total stomach contents})}{(\kappa t_l + 1 - \exp(-\kappa t_0))\phi} = \frac{hW^m}{\phi + Q} \qquad \text{(vi-}a\text{)}$$

$$\frac{\kappa * (\text{weight of total stomach contents})}{(\kappa t_l + 1 - \exp(-\kappa t_{00}))\phi} = \frac{hW^m}{\phi + Q}. \qquad \text{(vi-}b\text{)}$$

Examples

The use of the formulas will now be demonstrated by means of constructed examples. The formulas were all given in a discrete formulation, but from now on a continuous formulation is used, summing over prey species and integrating over weight ranges. The continuous form of (iii-*a*) is

$$\widetilde{N}_{st}\,dw\,\frac{\kappa}{\kappa t_l+\kappa t_0} = w\widetilde{N}_{st}\,d\log w\,\frac{\kappa}{\kappa t_l+\kappa t_0}$$

$$= \frac{\rho\exp(-(\log\frac{W}{w}-\eta)^2/2/\sigma^2)\,wN\,d\log w\;hW^m}{\phi+Q}$$

$$= \rho\exp(-(\log\frac{W}{w}-\eta)^2/2/\sigma^2)\,wN\,d\log w$$

$$* \frac{\kappa(\text{weight of recognizable part of total stomach contents})}{\phi(\kappa t_l+1-\exp(-\kappa t_0))}$$

$$= \rho\exp(-(\log\frac{W}{w}-\eta)^2/2/\sigma^2)\,wN\,d\log w$$

$$* \frac{\kappa(\text{weight of total stomach contents})}{\phi(\kappa t_l+1-\exp(-\kappa t_{00}))} \qquad \text{(iii-}b\text{)}$$

as found from (iii-*a*).

In all examples we have one predator species *Pr* feeding on several prey species *pr*1, *pr*2, etc. The digestion parameter κ depends only on W_{Pr} and is given as

$$\kappa = \kappa_0 W_{Pr}^{\kappa_1}.$$

The halfsaturation parameter Q is of the same form

$$Q = qW^r$$

m is assumed known and to be 0.56 and the ρ's are assumed independent of W.

Example 1

Two feeding experiments have given data for four *Pr* sizes

10 g 40 g 160 g 640 g

Experiment 1:

The *Pr*'s were feeding on four prey species

*pr*1 *pr*2 *pr*3 *pr*4

with environmental distributions:

$$N_{pr}(w)\,dw = wN_{pr}(w)\,d\log w = a_{pr}N_0\,d\log w$$

$a_{pr1} = 800$; $a_{pr2} = 400$; $a_{pr3} = 300$; $a_{pr4} = 25$;

i.e. distributions with the same number of animals in each log weight class.

In table 1 are given $w\widetilde{N}_{st}\mathrm{dlog}w$, the prey numbers per *Pr* stomach for each ingestion weight class. Total stomach content, *stctt*, and recognizable stomach content, *stctr*, are also given.

Determination of η and σ^2:

The 'available' food is

$$\phi = \sum_{pr} \int_{-\infty}^{+\infty} \rho_{pr}\exp\{-(\log(W/w) - \eta_{pr})^2/2/\sigma^2_{pr}\}\; w a_{pr} N_0 \mathrm{dlog}w.$$

Using the substitution $u = W/w$ we get

$$\phi = N_0 W \sum_{pr} \rho_{pr} a_{pr} \int_{-\infty}^{+\infty} \exp\{-(\log u - \eta_{pr})^2/2/\sigma^2_{pr}\} \frac{\mathrm{dlog}u}{u} = N_0 C_0 W.$$

The sum is a constant C_0 independent of W and this gives

$$w\widetilde{N}_{st}\mathrm{dlog}w \frac{W}{stctr} = \frac{\rho_{pr}a_{pr}\mathrm{dlog}w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)} \exp(-(\log u - \eta_{pr})^2/2/\sigma^2_{pr})$$

$$w\widetilde{N}_{st}\mathrm{dlog}w \frac{W}{stctt} = \frac{\rho_{pr}a_{pr}\mathrm{dlog}w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)} \exp(-(\log u - \eta_{pr})^2/2/\sigma^2_{pr})$$

where we have used (iii*b*).

This means that stomach numbers for all *Pr* sizes can be transformed to a single parabola in $\log u$ by multiplying by

$$\frac{W}{stctt} \quad \text{or} \quad \frac{W}{stctr}$$

and taking logs.

Parabolas have been fitted by least squares. The formula for the parabolas is

$$\psi(\log u) = \log\left\{\frac{\rho_{pr}a_{pr}\mathrm{dlog}w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr}}{2\sigma^2_{pr}} + \frac{\eta_{pr}}{\sigma^2_{pr}}\log u - \frac{1}{2\sigma^2_{pr}}\log^2 u$$

if *stctr* has been used and

$$\psi(\log u) = \log\left\{\frac{\rho_{pr}a_{pr}\mathrm{dlog}w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr}}{2\sigma^2_{pr}} + \frac{\eta_{pr}}{\sigma^2_{pr}}\log u - \frac{1}{2\sigma^2_{pr}}\log^2 u$$

if *stctt* has been used.

The four prey species give the results:
Using *stctr*

pr1: $\psi(\log u) = -18.8804 + 6.4783\log u - 0.4627\log^2 u$
$\sigma^2_{pr1} = 1/2/0.4627 = 1.0806.\ \eta_{pr1} = 6.4783/2/0.4627 = 7.0005$

pr2: $\psi(\log u) = -5.4220 + 2.8816\log u - 0.2401\log^2 u$
$\sigma^2_{pr2} = 1/2/0.2401 = 2.0825.\ \eta_{pr2} = 2.8816/2/0.2401 = 6.0008$

pr3: $\psi(\log u) = -11.4967 + 5.6727\log u - 0.5673\log^2 u$
$\sigma^2_{pr3} = 1/2/0.5673 = 0.8814.\ \eta_{pr3} = 5.6727/2/0.5673 = 4.9997$

pr4: $\psi(\log u) = -16.5697 + 8.2147\log u - 1.0267\log^2 u$
$\sigma^2_{pr4} = 1/2/1.0267 = 0.4870.\ \eta_{pr4} = 8.2147/2/1.0267 = 4.0005.$

Using *stctt*

pr1: $\psi(\log u) = -19.1959 + 6.4783\log u - 0.4627\log^2 u$
$\sigma^2_{pr1} = 1.0806.\ \eta_{pr1} = 7.0005$

pr2: $\psi(\log u) = -5.7374 + 2.8816\log u - 0.2401\log^2 u$
$\sigma^2_{pr2} = 2.0825.\ \eta_{pr2} = 6.0008.$

pr3: $\psi(\log u) = -11.8122 + 5.6727\log u - 0.5673\log^2 u$
$\sigma^2_{pr3} = 0.8814.\ \eta_{pr3} = 4.9997.$

pr4: $\psi(\log u) = -16.8851 + 8.2147\log u - 1.0267\log^2 u$
$\sigma^2_{pr4} = 0.4870.\ \eta_{pr4} = 4.0005.$

Determination of ρ:
The intercept for ψ is

$$\text{intercept}_{pr} = \log\left\{\frac{\rho_{pr}\, a_{pr}\, \mathrm{d}\log w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr}}{2\sigma^2_{pr}}$$

giving

$$\log\left\{\frac{\rho_{pr1}\ 800\ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)}\right\} = -18.8804 + 6.4783^2/4/0.4627 = 3.7954$$

$$\log\left\{\frac{\rho_{pr1}\ 800\ \ 1}{C_0\left(\dfrac{\kappa_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_{00}}\right)}\right\} = -19.1959 + 6.4783^2/4/0.4627 = 3.4799$$

$$\log\left\{\frac{\rho_{pr2}\ 400\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)}\right\} = -5.4220 + 2.8816^2/4/0.2401 = 3.2240$$

$$\log\left\{\frac{\rho_{pr2}\,400\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} = -5.7374 + 2.8816^2/4/0.2401 = 2.9086$$

$$\log\left\{\frac{\rho_{pr3}\,300\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)}\right\} = -11.4967 + 5.6727^2/4/0.5673 = 2.6843$$

$$\log\left\{\frac{\rho_{pr3}\,300\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} = -11.8122 + 5.6727^2/4/0.5673 = 2.3688$$

$$\log\left\{\frac{\rho_{pr4}\ 25\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right)}\right\} = -16.5697 + 8.2147^2/4/1.0267 = -0.1381$$

$$\log\left\{\frac{\rho_{pr4}\ 25\ \ 1}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} = -16.8851 + 8.2147^2/4/1.0267 = -0.4535.$$

We now take ρ_{pr2} as 1 and get

$$C_0\left(\frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0}\right) = 400/\exp(3.2240) = 15.918$$

$$C_0\left(\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right) = 400/\exp(2.9086) = 21.821$$

and

$$\rho_{pr1} = \exp(3.7954)\ 15.918/800 = \exp(3.4799)\ 21.821/800 = 0.8854$$
$$\rho_{pr3} = \exp(2.6843)\ 15.918/300 = \exp(2.3688)\ 21.821/300 = 0.7772$$
$$\rho_{pr4} = \exp(-0.1381)\ 15.918/25 = \exp(-0.4535)\ 21.821/25 = 0.5546$$
$$\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + 1 - \exp(-\kappa t_0)} = 21.821/15.918 = 1.3708.$$

Determination of $\widetilde{\kappa}_0 = \dfrac{\kappa_0}{h(\kappa t_l + 1 - \exp(\kappa t_{0(0)}))}$, κ_1, $q\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}$ *and* r:

(vi-$a(b)$) gives for N_0 chosen as 1

$$\widetilde{\kappa}_0 W^{\kappa_1 - m} + \frac{\widetilde{\kappa}_0 q}{C_0} W^{r+\kappa_1-1-m} = \frac{1}{stctr(t)}$$

and the four Pr sizes give the nonlinear equations

$$\widetilde{\kappa}_0\, 10^{\kappa_1 - m} + \frac{\widetilde{\kappa}_0 q}{C_0}\, 10^{r+\kappa_1-1-m} = 30.754\ (22.434)$$

$$\widetilde{\kappa}_0\, 40^{\kappa_1 - m} + \frac{\widetilde{\kappa}_0\, q}{C_0}\, 40^{r+\kappa_1-1-m} = 4.7402\ (3.4577)$$

$$\widetilde{\kappa}_0\, 160^{\kappa_1 - m} + \frac{\widetilde{\kappa}_0\, q}{C_0}\, 160^{r+\kappa_1-1-m} = 1.1943\ (0.87116)$$

$$\widetilde{\kappa}_0\, 640^{\kappa_1 - m} + \frac{\widetilde{\kappa}_0\, q}{C_0}\, 640^{r+\kappa_1-1-m} = 0.36556\ (0.26667)$$

with the solutions

$$\widetilde{\kappa}_0 = \frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))} = 67.448\ (49.181)$$

$$\kappa_1 - m = -0.8100\ (-0.8099)$$

$$\frac{\kappa_0\, q}{C_0 h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))} = 1851.9\ (1350.7)$$

$$r + \kappa_1 - 1 - m = -1.9599\ (-1.9598)$$

and we find

$$\kappa_1 = -0.2500\ (-0.2499)$$

$$q\, \frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 1851.9\ 15.918/67.448 = 437.06$$

$$q\, \frac{\kappa t_l + 1 - \exp(-\, t_{00})}{\kappa t_l + \kappa t_0} = 1350.7\ 21.821/49.181 = 599.29$$

$$r = -0.1499\ (-0.1498).$$

Determination of feeding level f:

$$f = C_0 W N_0 / (C_0 W N_0 + q W^r)$$

$$= \frac{C_0 \dfrac{\kappa t_l + 1 + \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0} W N_0}{C_0 \dfrac{\kappa t_l + 1 + \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0} W N_0 + q \dfrac{\kappa t_l + 1 + \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0} W^r}$$

or

$f_{10} = 15.918\ 10\ 1/(15.918\ 10\ 1 + 437.06\ 10^{-0.1499}) = 0.3396$
$(= 21.821\ 10\ 1/(21.821\ 10\ 1 + 599.28\ 10^{-0.1498}) = 0.3395)$
$f_{40} = 0.7169\ (0.7168)$
$f_{160} = 0.9258\ (0.9257)$
$f_{640} = 0.9840\ (0.9840).$

Table 1. Example 1, experiment 1. Stomach data.

dlog$w$ = 1.

stctr(10) = 3.2516E−2. *stctt*(10) = 4.4575E−2.
stctr(40) = 2.1096E−1. *stctt*(40) = 2.8921E−1.
stctr(160) = 8.3731E−1. *stctt*(160) = 1.1479.
stctr(640) = 2.7355. *stctt*(640) = 3.7500.

$\Omega = W/stctr$. $\Omega_1 = W/stctt$.

*pr*1

W	log$w$	log(W/w)	$w\tilde{N}_{st}$ dlog$w$	log $\Omega w\tilde{N}_{st}$ dlog$w$	log $\Omega_1 w\tilde{N}_{st}$ dlog$w$
	−7.5	9.8026	3.821E−3	0.1614	−0.1540
	−6.5	8.8026	3.220E−2	2.2927	1.9772
	−5.5	7.8026	1.070E−1	3.4934	3.1779
10	−4.5	6.8026	1.413E−1	3.7717	3.4562
	−3.5	5.8026	7.436E−2	3.1298	2.8144
	−2.5	4.8026	1.552E−2	1.5630	1.2475
	−1.5	3.8026	1.272E−3	−0.9389	−1.2543
	−6.5	10.1889	2.116E−3	−0.9135	−1.2290
	−5.5	9.1889	2.561E−2	1.5803	1.2648
	−4.5	8.1889	1.218E−1	3.1392	2.8238
40	−3.5	7.1889	2.295E−1	3.7732	3.4577
	−2.5	6.1889	1.724E−1	3.4870	3.1716
	−1.5	5.1889	5.148E−2	2.2785	1.9630
	−0.5	4.1889	6.062E−3	0.1392	−0.1763
	−5.5	10.5752	6.221E−4	−2.1297	−2.4452
	−4.5	9.5752	1.084E−2	0.7284	0.4130
	−3.5	8.5752	7.386E−2	2.6471	2.3317
160	−2.5	7.5752	1.988E−1	3.6375	3.3220
	−1.5	6.5752	2.131E−1	3.7066	3.3912
	−0.5	5.5752	9.093E−2	2.8551	2.5396
	0.5	4.5752	1.536E−2	1.0766	0.7611
	1.5	3.5752	1.016E−3	−1.6395	−1.9550
	−3.5	9.9615	3.284E−3	−0.2636	−0.5791
	−2.5	8.9615	3.211E−2	2.0165	1.7011
	−1.5	7.9615	1.236E−1	3.3644	3.0489
640	−0.5	6.9615	1.890E−1	3.7889	3.4735
	0.5	5.9615	1.151E−1	3.2934	2.9780
	1.5	4.9615	2.785E−2	1.8741	1.5587
	2.5	3.9615	2.649E−3	−0.4784	−0.7938

pr2

W	$\log w$	$\log(W/w)$	$w\widetilde{N}_{st}\,d\log w$	$\log\ \Omega w\widetilde{N}_{st}\,d\log w$	$\log\ \Omega_1 w\widetilde{N}_{st}\,d\log w$
	−7.5	9.8026	2.535E−3	−0.2488	−0.5642
	−6.5	8.8026	1.239E−2	1.3377	1.0223
	−5.5	7.8026	3.741E−2	2.4429	2.1274
	−4.5	6.8026	6.988E−2	3.0677	2.7522
10	−3.5	5.8026	8.080E−2	3.2128	2.8973
	−2.5	4.8026	5.783E−2	2.8783	2.5629
	−1.5	3.8026	2.561E−2	2.0640	1.7485
	−0.5	2.8026	7.015E−3	0.7689	0.4535
	0.5	1.8026	1.187E−3	−1.0080	−1.3234
	−6.5	10.1889	1.958E−3	−0.9908	−1.3062
	−5.5	9.1889	1.153E−2	0.7820	0.4666
	−4.5	8.1889	4.192E−2	2.0729	1.7575
	−3.5	7.1889	9.426E−2	2.8832	2.5678
40	−2.5	6.1889	1.312E−1	3.2136	2.8981
	−1.5	5.1889	1.130E−1	3.0644	2.7489
	−0.5	4.1889	6.024E−2	2.4355	2.1200
	0.5	3.1889	1.987E−2	1.3262	1.0108
	1.5	2.1889	4.049E−3	−0.2644	−0.5798
	2.5	1.1889	5.092E−4	−2.3377	−2.6531
	−5.5	10.5752	8.608E−4	−1.8049	−2.1203
	−4.5	9.5752	6.107E−3	0.1544	−0.1611
	−3.5	8.5752	2.674E−2	1.6313	1.3158
	−2.5	7.5752	7.240E−2	2.6271	2.3117
160	−1.5	6.5752	1.212E−1	3.1428	2.8274
	−0.5	5.5752	1.257E−1	3.1789	2.8634
	0.5	4.5752	8.067E−2	2.7353	2.4199
	1.5	3.5752	3.203E−2	1.8117	1.4963
	2.5	2.5752	7.863E−3	0.4072	0.0918
	3.5	1.5752	1.192E−3	−1.4794	−1.7949
	−3.5	9.9615	2.478E−3	−0.5452	−0.8607
	−2.5	8.9615	1.307E−2	1.1179	0.8024
	−1.5	7.9615	4.261E−2	2.2994	1.9839
	−0.5	6.9615	8.589E−2	3.0005	2.6850
640	0.5	5.9615	1.072E−1	3.2218	2.9063
	1.5	4.9615	8.277E−2	2.9635	2.6481
	2.5	3.9615	3.957E−2	2.2254	1.9100
	3.5	2.9615	1.170E−2	1.0068	0.6914
	4.5	1.9615	2.137E−3	−0.6934	−1.0089

Table 1 continued

pr3

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\,d\log w$	$\log$ $\Omega w\tilde{N}_{st}\,d\log w$	$\log$ $\Omega_1 w\tilde{N}_{st}\,d\log w$
10	−5.5	7.8026	5.499E−4	−1.7771	−2.0926
	−4.5	6.8026	7.574E−3	0.8456	0.5302
	−3.5	5.8026	3.303E−2	2.3183	2.0028
	−2.5	4.8026	4.644E−2	2.6590	2.3436
	−1.5	3.8026	2.116E−2	1.8729	1.5574
	−0.5	2.8026	3.089E−3	−0.0513	−0.3667
40	−3.5	7.1889	5.118E−3	−0.0300	−0.3454
	−2.5	6.1889	3.471E−2	1.8844	1.5689
	−1.5	5.1889	7.547E−2	2.6609	2.3454
	−0.5	4.1889	5.316E−2	2.3105	1.9951
	0.5	3.1889	1.207E−2	0.8281	0.5127
	1.5	2.1889	8.679E−4	−1.8045	−2.1200
160	−2.5	7.5752	1.780E−3	−1.0785	−1.3939
	−1.5	6.5752	1.884E−2	1.2808	0.9653
	−0.5	5.5752	6.342E−2	2.4948	2.1793
	0.5	4.5752	6.902E−2	2.5794	2.2639
	1.5	3.5752	2.431E−2	1.5359	1.2205
	2.5	2.5752	2.733E−3	−0.6497	−0.9652
640	−0.5	6.9615	7.092E−3	0.5063	0.1909
	0.5	5.9615	3.707E−2	2.1603	1.8448
	1.5	4.9615	6.235E−2	2.6801	2.3647
	2.5	3.9615	3.399E−2	2.0733	1.7579
	3.5	2.9615	5.953E−3	0.3313	0.0158

pr4

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\,d\log w$	$\log$ $\Omega w\tilde{N}_{st}\,d\log w$	$\log$ $\Omega_1 w\tilde{N}_{st}\,d\log w$
10	−2.5	4.8026	1.467E−3	−0.7957	−1.1112
	−1.5	3.8026	2.716E−3	−0.1799	−0.4954
	−0.5	2.8026	6.506E−4	−1.6091	−1.9245
40	−1.5	5.1889	1.078E−3	−1.5879	−1.9034
	−0.5	4.1889	4.420E−3	−0.1766	−0.4921
	0.5	3.1889	2.347E−3	−0.8098	−1.1252
160	0.5	4.5752	3.251E−3	−0.4760	−0.7915
	1.5	3.5752	3.788E−3	−0.3233	−0.6387
	2.5	2.5752	5.629E−4	−2.2297	−2.5451
640	1.5	4.9615	1.447E−3	−1.0832	−1.3986
	2.5	3.9615	3.708E−3	−0.1420	−0.4575
	3.5	2.9615	1.235E−3	−1.2419	−1.5573

Experiment 2:
The second experiment is similar to the first. The differences are:
Pr is preying on 6 *pr* species with

$$a_{pr1} = 1.\ a_{pr2} = 10.\ a_{pr3} = 0.1.\ a_{pr4} = 1.\ a_{pr5} = 1.\ a_{pr6} = 0.1.$$

f is known to be 1 for all *Pr* sizes.
Data for *pr*1 and *pr*2 are shown in table 2.

Determination of $\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))$ *and* κ_1:
(v-$a(b)$) gives

$$stctr(t) = \log(stctr(t)) = -\log(\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))) + (m - \kappa_1)\log W$$

or

$$\log(stctr(t)) = -\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{0(0)})) + (m - \kappa_1)\log W$$

a linear regression in $\log W$ and the data give

$$\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{0(0)})) = 67.464\ (49.205)$$
$$m - \kappa_1 = 0.8100\ (0.8100)$$
$$\kappa_1 = -0.2500\ (-0.2500).$$

Determination of η *and* σ^2:
The stomach numbers can be placed on single parabolas by using two sets of weighting factors

$$W/stctr(t)$$

as in the first experiment and

$$W^{1-m+\kappa_1}.$$

The two parabola formulas are

$$\log\left(w\widetilde{N}_{st}\,\mathrm{d}\log w\,\frac{W}{stctr(t)}\right)$$
$$= \log\left\{\frac{\rho_{pr}\,a_{pr}\,\mathrm{d}\log w}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta_{pr}^2}{2\sigma_{pr}^2} + \frac{\eta_{pr}\log u}{\sigma_{pr}^2} - \frac{\log^2 u}{2\sigma_{pr}^2}$$

$$\log(w\widetilde{N}_{st}\,\mathrm{d}\log w\,W^{1-m+\kappa_1}) = \log\frac{h\rho_{pr}\,a_{pr}\,\mathrm{d}\log w}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}C_0} - \frac{\eta_{pr}^2}{2\sigma_{pr}^2} + \frac{\eta_{pr}\log u}{\sigma_{pr}^2} - \frac{\log^2 u}{2\sigma_{pr}^2}$$

where we have used (iii-b), the ϕ formula from the first experiment and $f = 1$.

Least squares gives

$$\psi(\log u) = -21.3578 + 6.4649 \log u - 0.4618 \log^2 u \qquad (pr1)$$
$$\sigma^2_{pr1} = 1.0827. \ \eta_{pr1} = 6.9997$$

$$\psi(\log u) = -4.9500 + 2.8822 \log u - 0.2402 \log^2 u \qquad (pr2)$$
$$\sigma^2_{pr2} = 2.0816. \ \eta_{pr2} = 5.9996.$$

using *W/stctr*, and

$$\psi(\log u) = -21.6732 + 6.4649 \log u - 0.4618 \log^2 u \qquad (pr1)$$
$$\sigma^2_{pr1} = 1.0827. \ \eta_{pr1} = 6.9997$$

$$\psi(\log u) = -5.2655 + 2.8822 \log u - 0.2402 \log^2 u \qquad (pr2)$$
$$\sigma^2_{pr2} = 2.0816. \ \eta_{pr} = 5.9996$$

using *W/stctt*, and

$$\psi(\log u) = -25.5692 + 6.4649 \log u - 0.4618 \log^2 u \qquad (pr1)$$
σ^2 and η as before

$$\psi(\log u) = -9.1614 + 2.8822 \log u - 0.2402 \log^2 u \qquad (pr2)$$
σ^2 and η as before

using $W^{1-m+\kappa_1}$.

Determination of ρ:

We take ρ_{pr2} as 1 again and get

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 10 \exp(4.9500 - 2.8822^2/4/0.2402) = 0.24823$$

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 10 \exp(5.2655 - 2.8822^2/4/0.2402) = 0.34030$$

$$C_0 \frac{\kappa_0}{h(\kappa t_l + \kappa t_0)} = 10 \exp(9.1614 - 2.8822^2/4/0.2402) = 16.743$$

and from this

$$\rho_{pr1} = \exp(-21.3578 + 6.4649^2/4/0.4618)\ 0.24823 = 0.8824$$
$$\rho_{pr1} = \exp(-21.6732 + 6.4649^2/4/0.4618)\ 0.34030 = 0.8824$$
$$\rho_{pr1} = \exp(-25.5692 + 6.4649^2/4/0.4618)\ 16.743 = 0.8824.$$

We also get the following controls:

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_0))} = 16.743/0.24823 = 67.450$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))} = 16.743/0.34030 = 49.201.$$

The only parameters left are q and r but as $f = 1$ it is impossible to determine these.

Table 2: Example 1, experiment 2. Stomach data.

$\text{dlog}w = 1$.

$stctr(10) = 9.5716\text{E}{-2}$. $stctt(10) = 1.3122\text{E}{-1}$.
$stctr(40) = 2.9424\text{E}{-1}$. $stctt(40) = 4.0336\text{E}{-1}$.
$stctr(160) = 9.0443\text{E}{-1}$. $stctt(160) = 1.2399$.
$stctr(640) = 2.7800$. $stctt(640) = 3.8110$. $\Omega = W/stctr$. $\Omega_1 = W/stctt$. $\Omega_2 = W^{1-m+\kappa_1}$.

*pr*1

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\text{dlog}w$	log $\Omega w\tilde{N}_{st}\text{dlog}w$	log $\Omega_1 w\tilde{N}_{st}\text{dlog}w$	log $\Omega_2 w\tilde{N}_{st}\text{dlog}w$
10	−7.5	9.8026	9.031E−4	−2.3608	−2.6762	−6.5722
	−6.5	8.8026	7.609E−3	−0.2295	−0.5449	−4.4410
	−5.5	7.8026	2.528E−2	0.9712	0.6557	−3.2403
	−4.5	6.8026	3.339E−2	1.2495	0.9340	−2.9620
	−3.5	5.8026	1.757E−2	0.6077	0.2922	−3.6038
	−2.5	4.8026	3.668E−3	−0.9592	−1.2746	−5.1707
40	−5.5	9.1889	2.868E−3	−0.9419	−1.2573	−5.1532
	−4.5	8.1889	1.363E−2	0.6170	0.3016	−3.5943
	−3.5	7.1889	2.570E−2	1.2510	0.9356	−2.9603
	−2.5	6.1889	1.930E−2	0.9648	0.6494	−3.2465
	−1.5	5.1889	5.765E−3	−0.2437	−0.5592	−4.4551
	−0.5	4.1889	6.788E−4	−2.3830	−2.6985	−6.5944
160	−4.5	9.5752	9.402E−4	−1.7938	−2.1092	−6.0051
	−3.5	8.5752	6.405E−3	0.1249	−0.1905	−4.0864
	−2.5	7.5752	1.724E−2	1.1153	0.7998	−3.0960
	−1.5	6.5752	1.848E−2	1.1844	0.8690	−3.0269
	−0.5	5.5752	7.885E−3	0.3329	0.0174	−3.8785
	0.5	4.5752	1.332E−3	−1.4456	−1.7610	−5.6569
640	−2.5	8.9615	2.620E−3	−0.5056	−0.8211	−4.7170
	−1.5	7.9615	1.008E−2	0.8422	0.5267	−3.3692
	−0.5	6.9615	1.542E−2	1.2667	0.9513	−2.9446
	0.5	5.9615	9.393E−3	0.7712	0.4558	−3.4401
	1.5	4.9615	2.272E−3	−0.6481	−0.9635	−4.8594

*pr*2

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\text{dlog}w$	log $\Omega w\tilde{N}_{st}\text{dlog}w$	log $\Omega_1 w\tilde{N}_{st}\text{dlog}w$	log $\Omega_2 w\tilde{N}_{st}\text{dlog}w$
10	−8.5	10.8026	1.514E−3	−1.8444	−2.1598	−6.0558
	−7.5	9.8026	1.198E−2	0.2248	−0.0907	−3.9867
	−6.5	8.8026	5.856E−2	1.8113	1.4958	−2.4002
	−5.5	7.8026	1.768E−1	2.9164	2.6010	−1.2950
	−4.5	6.8026	3.303E−1	3.5412	3.2258	−0.6702
	−3.5	5.8026	3.819E−1	3.6863	3.3709	−0.5251
	−2.5	4.8026	2.733E−1	3.3519	3.0364	−0.8596
	−1.5	3.8026	1.211E−1	2.5375	2.2221	−1.6740
	−0.5	2.8026	3.316E−2	1.2425	0.9270	−2.9690
	0.5	1.8026	5.609E−3	−0.5344	−0.8499	−4.7459
	1.5	0.8026	5.852E−4	−2.7946	−3.1100	−7.0060

Table 2 continued

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\,d\log w$	$\log\ \Omega w\tilde{N}_{st}\,d\log w$	$\log\ \Omega_1 w\tilde{N}_{st}\,d\log w$	$\log\ \Omega_2 w\tilde{N}_{st}\,d\log w$
	−6.5	10.1889	4.385E−3	−0.5172	−0.8327	−4.7286
	−5.5	9.1889	2.582E−2	1.2556	0.9401	−2.9558
	−4.5	8.1889	9.388E−2	2.5465	2.2310	−1.6649
	−3.5	7.1889	2.111E−1	3.3568	3.0413	−0.8546
40	−2.5	6.1889	2.937E−1	3.6871	3.3717	−0.5242
	−1.5	5.1889	2.530E−1	3.5379	3.2225	−0.6734
	−0.5	4.1889	1.349E−1	2.9090	2.5936	−1.3023
	0.5	3.1889	4.449E−2	1.7998	1.4843	−2.4116
	1.5	2.1889	9.067E−3	0.2092	−0.1063	−4.0022
	2.5	1.1889	1.140E−3	−1.8641	−2.1796	−6.0755
	−5.5	10.5752	1.493E−3	−1.3313	−1.6468	−5.5427
	−4.5	9.5752	1.059E−2	0.6279	0.3125	−3.5834
	−3.5	8.5752	4.638E−2	2.1048	1.7894	−2.1065
	−2.5	7.5752	1.256E−1	3.1007	2.7852	−1.1107
160	−1.5	6.5752	2.103E−1	3.6164	3.3009	−0.5950
	−0.5	5.5752	2.180E−1	3.6524	3.3370	−0.5589
	0.5	4.5752	1.399E−1	3.2089	2.8934	−1.0025
	1.5	3.5752	5.556E−2	2.2853	1.9698	−1.9261
	2.5	2.5752	1.364E−2	0.8808	0.5653	−3.3306
	3.5	1.5752	2.067E−3	−1.0059	−1.3214	−5.2172
	−3.5	9.9615	4.043E−3	−0.0717	−0.3871	−4.2830
	−2.5	8.9615	2.133E−2	1.5914	1.2760	−2.6199
	−1.5	7.9615	6.953E−2	2.7730	2.4575	−1.4384
	−0.5	6.9615	1.402E−1	3.4740	3.1586	−0.7373
640	0.5	5.9615	1.749E−1	3.6953	3.3799	−0.5160
	1.5	4.9615	1.351E−1	3.4371	3.1216	−0.7743
	2.5	3.9615	6.457E−2	2.6990	2.3835	−1.5123
	3.5	2.9615	1.909E−2	1.4804	1.1649	−2.7310
	4.5	1.9615	3.486E−3	−0.2199	−0.5353	−4.4312

Example 2

The predator *Pr* has preyed on a single prey species *pr7* with environmental distributions of the form

$$N_{pr}(w)\,dw = wN_{pr}(w)\,d\log w = N_1 w^{-\nu}\,d\log w$$

and we have data from two environments.

Environment 1:

Table 3 gives stomach data for an environment with $\nu = 0.5$ for seven *Pr* weights

10 g, 20 g, 40 g, 80 g, 160 g, 320 g, 640 g.

Determination of σ^2 and η:

The 'available' food ϕ is

$$\phi = \int_{-\infty}^{\infty} \rho_{pr7}\exp(-\,(\log(W_{Pr}/w) - \eta_{pr7})^2/2/\sigma^2_{pr7})\,wN_1 w^{-\nu}\,d\log w$$

and the substitution $u = W/w$ gives

$$\phi = N_1 W^{1-\nu}_{Pr} \rho_{pr7} \int_{-\infty}^{+\infty} \exp(-(\log u - \eta_{pr7})^2/2/\sigma^2_{pr7}) u^{\nu-1} d\log u = N_1 W^{1-\nu}_{Pr} C_1 .$$

(iii-b) can now be written in the following ways

$$w\widetilde{N}_{st}\, d\log w \frac{W_{Pr}}{stctr(t)} = \frac{\rho_{pr7}\, d\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)} \exp(-(\log u - \eta_{pr7})^2/2/\sigma^2_{pr7})^2 u^{\nu}$$

and

$$w\widetilde{N}_{st}\, d\log w \frac{W^{1-\nu}_{Pr} w^{\nu}}{stctr(t)} = \frac{\rho_{pr7}\, d\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)} \exp(-(\log u - \eta_{pr7})^2/2/\sigma^2_{pr7}).$$

Stomach numbers are now transformed to parabolas by means of the factors

$\Omega = W_{Pr}/stctr(t)$

$\Omega_1 = W^{1-\nu}_{Pr}\ w^{\nu}/stctr(t)$

and taking logs:

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\, d\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{(\eta_{pr7} + \sigma^2_{pr7}\nu)}{\sigma^2_{pr7}} \log u - \frac{\log^2 u}{2\sigma^2_{pr7}}$$

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\, d\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{\eta_{pr7}}{\sigma^2_{pr7}} \log u - \frac{\log^2 u}{2\sigma^2_{pr7}} .$$

Least squares gives

$\psi(\log u) = -11.9676 + 5.5549 \log u - 0.4629 \log^2 u$

$\sigma^2_{pr7} = 1.0801.\ \eta_{pr7} + \sigma^2_{pr7}\nu = 6.0001.\ \eta_{pr7} = 5.4600.$ (Ω and *stctr*)

$\psi(\log u) = -12.2831 + 5.5549 \log u - 0.4629 \log^2 u$ (*stctt* and Ω)

σ^2 and η as for *stctr*.

$\psi(\log u) = -11.9676 + 5.0549 \log u - 0.4629 \log^2 u$

$\sigma^2_{pr7} = 1.0801.\ \eta_{pr7} = 5.4600$ (Ω_1 and *stctr*).

$\psi(\log u) = -12.2831 + 5.0549 \log u - 0.4629 \log^2 u$ (*stctt* and Ω_1)

σ^2 and η as for *stctr*.

Determination of $\widetilde{\kappa}_0 = \kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))$, κ_1, $q(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))/(\kappa t_l + \kappa t_0)/N_1$ *and* r:

(vi-$a(b)$) gives:

$$\widetilde{\kappa}_0 W^{-m+\kappa_1} + \widetilde{\kappa}_0 \frac{q}{C_1 N_1} W^{r-m+\kappa_1-1+\nu} = 1/stctr(t)$$

which gives us an equation for each *Pr* weight. Calculations show that

$\widetilde{\kappa}_0 = 67.409(49.149)$ $\quad -m+\kappa_1 = -0.8099(-0.8099)$

$\widetilde{\kappa}_0 q/C_1 N_1 = 928.50(677.24)$ $\quad r-m+\kappa_1-1+\nu = -1.4599(-1.4598)$

give a nice fit.

Taking $\rho_{pr7} = 1$ the ψ intercept is

$$\text{intercept} = -\log\left(C_1 \frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right) - \frac{\eta_{pr}^2}{2\sigma_{pr}^2}$$

and we find

$C_1(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))/(\kappa t_l + \kappa t_0) = \exp(-11.9676(-12.2831) + 5.0549/4/0.4629) = 0.16003(0.21940).$

which gives us

$$\kappa_1 = -0.2499(-0.2499)$$

$$\frac{q(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}{(\kappa t_l + \kappa t_0)N_1} = 928.50\ 0.16003/67.409 = 2.2043(3.0232)$$

$$r = -0.1500(-0.1499).$$

Determination of feeding level f:
The feeding levels are now determined from

$$fw = \frac{C_1 W^{1-\nu}}{C_1 W^{1-\nu} + \frac{q}{N_1} W^r}$$

$$= \frac{C_1\left(\frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right) W^{1-\nu}}{C_1\left(\frac{\kappa t_l + 1 + \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right) W^{1-\nu} + \frac{q}{N_1}\left(\frac{\kappa t_l + 1 + \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right) W^r}$$

and we get

$f_{10} = 0.2449(0.2448)$
$f_{20} = 0.3372(0.3371)$
$f_{40} = 0.4440(0.4438)$
$f_{80} = 0.5561(0.5559)$
$f_{160} = 0.6629(0.6627)$
$f_{320} = 0.7552(0.7550)$
$f_{640} = 0.8288(0.8287)$.

Table 3: Example 2, environment 1. Stomach data.

dlog$w$ = 1.

stctr(10) = 2.3450E−2. *stctt*(10) = 3.2147E−2. *stctr*(20) = 56618E−2. *stctt*(20) = 7.7616E−2. *stctr*(40) = 1.3067E−1. *stctt*(40) = 1.7913E−1. *stctr*(80) = 2.8695E−1. *stctt*(80) = 3.9338E−1. *stctr*(160) = 5.9960E−1. *stctt*(160) = 8.2198E−1. *stctr*(320) = 1.1976. *stctt*(320) = 1.6418. *stctr*(640) = 2.3042. *stctt*(640) = 3.1588.

$\Omega_r = W/stctr$. $\Omega_t = W/stctt$. $\Omega_{1r} = W^{1-\nu}w^{\nu}/stctr$. $\Omega_{1t} = W^{1-\nu}w^{\nu}/stctt$.

W	log$w$	log(W/w)	$w\tilde{N}_{st}$ dlog$w$	$\log(\Omega_r w\tilde{N}_{st} \mathrm{dlog}w)$	$\log(\Omega_t w\tilde{N}_{st} \mathrm{dlog}w)$	$\log(\Omega_{1r} w\tilde{N}_{st} \mathrm{dlog}w)$	$\log(\Omega_{1t} w\tilde{N}_{st} \mathrm{dlog}w)$
	−6.5	8.8026	6.803E−3	1.0651	0.7496	−3.3362	−3.6517
	−5.5	7.8026	5.732E−2	3.1964	2.8809	−0.7049	−1.0204
	−4.5	6.8026	1.904E−1	4.3970	4.0816	0.9957	0.6803
10	−3.5	5.8026	2.515E−1	4.6754	4.3599	1.7741	1.4586
	−2.5	4.8026	1.324E−1	4.0335	3.7181	1.6322	1.3168
	−1.5	3.8026	2.763E−2	2.4667	2.1512	0.5654	0.2499
	−0.5	2.8026	2.264E−3	−0.0352	−0.3506	−1.4365	−1.7519
	−6.5	9.4957	1.079E−3	−0.9644	−1.2798	−5.7122	−6.0277
	−5.5	8.4957	1.745E−2	1.8187	1.5033	−2.4291	−2.7446
	−4.5	7.4957	1.104E−1	3.6633	3.3478	−0.0846	−0.4000
20	−3.5	6.4957	2.761E−1	4.5803	4.2648	1.3324	1.0170
	−2.5	5.4957	2.751E−1	4.5764	4.2609	1.8285	1.5130
	−1.5	4.4957	1.091E−1	3.6515	3.3360	1.4036	1.0882
	−0.5	3.4957	1.711E−2	1.7990	1.4835	0.0511	−0.2644
	0.5	2.4957	1.050E−3	−0.9922	−1.3076	−2.2401	−2.5555
	−5.5	9.1889	3.235E−3	−0.0098	−0.3253	−4.6043	−4.9197
	−4.5	8.1889	3.917E−2	2.4840	2.1685	−1.6105	−1.9259
	−3.5	7.1889	1.862E−1	4.0429	3.7274	0.4485	0.1330
40	−2.5	6.1889	3.510E−1	4.6769	4.3614	1.5824	1.2670
	−1.5	5.1889	2.636E−1	4.3907	4.0752	1.7963	1.4808
	−0.5	4.1889	7.872E−2	3.1821	2.8667	1.0877	0.7722
	0.5	3.1889	9.269E−3	1.0429	0.7274	−0.5516	−0.8670
	−4.5	8.8820	8.439E−3	0.8555	0.5401	−3.5855	−3.9009
	−3.5	7.8820	7.660E−2	3.0612	2.7458	−0.8798	−1.1952
	−2.5	6.8820	2.739E−1	4.3355	4.0200	0.8944	0.5790
80	−1.5	5.8820	3.892E−1	4.6869	4.3714	1.7459	1.4304
	−0.5	4.8820	2.204E−1	4.1182	3.8028	1.6772	1.3618
	0.5	3.8820	4.952E−2	2.6252	2.3097	0.6842	0.3687
	1.5	2.8820	4.373E−3	0.1980	−0.1174	−1.2430	−1.5584
	−4.5	9.5752	1.100E−3	−1.2261	−1.5415	−6.0136	−6.3291
	−3.5	8.5752	1.917E−2	1.6321	1.3166	−2.6555	−2.9709
	−2.5	7.5752	1.306E−1	3.5508	3.2353	−0.2368	−0.5522
160	−1.5	6.5752	3.515E−1	4.5412	4.2257	1.2536	0.9381
	−0.5	5.5752	3.767E−1	4.6103	4.2949	1.8227	1.5073
	0.5	4.5752	1.607E−1	3.7588	3.4433	1.4712	1.1557
	1.5	3.5752	2.715E−2	1.9803	1.6648	0.1927	−0.1228
	2.5	2.5752	1.795E−3	−0.7359	−1.0513	−2.0234	−2.3389

Table 3 continued

W	log$w$	log(W/w)	$w\tilde{N}_{st}$dlog$w$	log($\Omega_r w\tilde{N}_{st}$dlog$w$)	log($\Omega_t w\tilde{N}_{st}$dlog$w$)	log($\Omega_{1r} w\tilde{N}_{st}$dlog$w$)	log($\Omega_{1t} w\tilde{N}_{st}$dlog$w$)
	−3.5	9.2683	2.919E−3	−0.2484	−0.5638	−4.8825	−5.1980
	−2.5	8.2683	3.809E−2	2.3202	2.0048	−1.8139	−2.1294
	−1.5	7.2683	1.950E−1	3.9530	3.6375	0.3188	0.0034
320	−0.5	6.2683	3.954E−1	4.6602	4.3447	1.5260	1.2106
	0.5	5.2683	3.195E−1	4.4471	4.1316	1.8129	1.4975
	1.5	4.2683	1.027E−1	3.3120	2.9966	1.1779	0.8624
	2.5	3.2683	1.303E−2	1.2471	0.9317	−0.3871	−0.7025
	3.5	2.2683	6.444E−4	−1.7592	−2.0746	−2.8933	−3.2088
	−2.5	8.9615	6.828E−3	0.6400	0.3246	−3.8407	−4.1561
	−1.5	7.9615	6.677E−2	2.9202	2.6048	−1.0605	−1.3760
	−0.5	6.9615	2.570E−1	4.2681	3.9526	0.7873	0.4719
640	0.5	5.9615	3.929E−1	4.6926	4.3771	1.7119	1.3964
	1.5	4.9615	2.394E−1	4.1971	3.8817	1.7164	1.4009
	2.5	3.9615	5.791E−2	2.7778	2.4623	0.7971	0.4816
	3.5	2.9615	5.509E−3	0.4253	0.1098	−1.0554	−1.3709

Environment 2:

Table 4 shows data from an environment similar to the first but it is known that

$$r = 1 - \nu$$

and

$$\nu = 1.15$$

Determination of κ_1.

$r = 1 - \nu$ means that f is independent of predator size W_{Pr} and we get

$$stctr(t) = \frac{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))f}{\kappa_0} W^{m-\kappa_1}$$

Linear regression gives:

$$m - \kappa_1 = 0.8100(0.8100) \text{ or } \kappa_1 = -0.2500(-0.2500)$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))f} = 96.602(70.464)\,.$$

Determination of σ^2 and η.

The stomach figures can be transformed to parabolas by means of the factors:

$W/stctr(t)$; $W^{1-\nu} w^{\nu}/stctr(t)$

$W^{1-m+\kappa_1}$; $W^{1-\nu-m+\kappa_1} w^{\nu}$.

The parabolas are

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\ \mathrm{d}\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{(\eta_{pr7} + \sigma^2_{pr7}\nu)}{\sigma^2_{pr7}}\log u - \frac{\log^2 u}{2\sigma^2_{pr7}}$$

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\ \mathrm{d}\log w}{C_1\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{\eta_{pr7}}{\sigma^2_{pr7}}\log u - \frac{\log^2 u}{2\sigma^2_{pr7}}$$

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\ \mathrm{d}\log w}{\dfrac{\kappa_0}{h(\kappa t_l + \kappa t_0)}\left(C_1 + \dfrac{q}{N_1}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{(\eta_{pr7} + \sigma^2_{pr7}\nu)}{\sigma^2_{pr7}}\log u - \frac{\log^2 u}{2\sigma^2_{pr7}}$$

$$\psi(\log u) = \log\left\{\frac{\rho_{pr7}\ \mathrm{d}\log w}{\dfrac{\kappa_0}{h(\kappa t_l + \kappa t_0)}\left(C_1 + \dfrac{q}{N_1}\right)}\right\} - \frac{\eta^2_{pr7}}{2\sigma^2_{pr7}} + \frac{\eta_{pr7}}{\sigma^2_{pr7}}\log u - \frac{\log^2 u}{2\sigma^2_{pr7}}\ .$$

The data from table 4 give:

$\psi(\log u) = -15.1400 + 6.1621\log u - 0.4633\log^2 u$
$\sigma^2_{pr7} = 1.0792.\ \ \eta_{pr7} + \sigma^2_{pr7}\nu = 6.6502.\ \ \eta_{pr7} = 5.4091.$ (*W/stctr*).
$\psi(\log u) = -15.4555 + 6.1621\log u - 0.4633\log^2 u.$ (*W/stctt*).
σ^2, $\eta + \sigma^2\nu$, and η as for *stctr*.
$\psi(\log u) = -15.1400 + 5.0121\log u - 0.4633\log^2 u$
$\sigma^2_{pr7} = 1.0792.\ \eta_{pr7} = 5.4091.$ ($W^{1-\nu} w^{\nu}/stctr$)
$\psi(\log u) = -15.4555 + 5.0121\log u - 0.4633\log^2 u.$ ($W^{1-\nu} w^{\nu}/stctt$).
σ^2, η as for *stctr*.
$\psi(\log u) = -19.7105 + 6.1621\log u - 0.4633\log^2 u.$ ($W^{1-m+\kappa_1}$)
σ^2 and η as for *W/stctr*.
$\psi(\log u) = -19.7105 + 5.0121\log u - 0.4633\log^2 u.$ ($W^{1-\nu-m+\kappa_1}w^{\nu}$)
σ^2, η as for $W^{1-\nu}w^{\nu}/stctr$.

The relation $r = 1 - \nu$ makes it impossible to determine κ_0, q, and f. The reason is that any choice that keeps

$$\kappa_0/f = \kappa_0(1 + q/N_1C_1)$$

constant will give the same stomach data.

The calculations in example 2 can be taken as a method for finding rough estimates of the mean parameters for all prey species if more than one species is present and the environmental distribution is of form

constant $w^{-\nu}$.

In example 3 we tackle the case of individual treatment of several species assuming that the form of 'available' food is as in example 2.

Table 4: Example 2, environment 2. Stomach data.

$d\log w = 1$.

$stctr(10) = 6.6840E-2$. $stctt(10) = 9.1630E-2$. $stctr(20) = 1.1719E-1$. $stctt(20) = 1.6065E-1$. $stctr(40) = 2.0545E-1$. $stctt(40) = 2.8165E-1$. $stctr(80) = 3.6020E-1$. $stctt(80) = 4.9379E-1$. $stctr(160) = 6.3150E-1$. $stctt(160) = 8.6572E-1$. $stctr(320) = 1.1072$. $stctt(320) = 1.5178$. $stctr(640) = 1.9411$. $stctt(640) = 2.6610$.

$\Omega_r = W/stctr$. $\Omega_t = W/stctt$. $\Omega_{1r} = W^{1-\nu}w^{\nu}/stctr$. $\Omega_{1t} = W^{1-\nu}w^{\nu}/stctt$. $\Omega_2 = W^{1-m+\kappa_1}$. $\Omega_3 = W^{1-\nu-m+\kappa_1}w^{\nu}$.

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}d\log w$	$\log(\Omega_r w\tilde{N}_{st}d\log w)$	$\log(\Omega_t w\tilde{N}_{st}d\log w)$	$\log(\Omega_{1r} w\tilde{N}_{st}d\log w)$	$\log(\Omega_{1t} w\tilde{N}_{st}d\log w)$	$\log(\Omega_2 w\tilde{N}_{st}d\log w)$	$\log(\Omega_3 w\tilde{N}_{st}d\log w)$
10	−7.5	9.8026	1.411E−2	0.7472	0.4317	−10.5258	−10.8413	−3.8234	−15.0963
	−6.5	8.8026	1.651E−1	3.2068	2.8914	−6.9161	−7.2316	−1.3637	−11.4867
	−5.5	7.8026	7.588E−1	4.7320	4.4166	−4.2409	−4.5564	0.1615	−8.8115
	−4.5	6.8026	1.383	5.3326	5.0171	−2.4904	−2.8058	0.7620	−7.0609
	−3.5	5.8026	1.005	5.0130	4.6975	−1.6600	−1.9754	0.4425	−6.2305
	−2.5	4.8026	2.902E−1	3.7708	3.4554	−1.7521	−2.0676	−0.7997	−6.3227
	−1.5	3.8026	3.303E−2	1.5976	1.2821	−2.7754	−3.0909	−2.9730	−7.3459
	−0.5	2.8026	1.464E−3	−1.5185	−1.8339	−4.7414	−5.0569	−6.0890	−9.3120
20	−7.5	10.4957	1.291E−3	−1.5124	−1.8279	−13.5825	−13.8980	−6.0830	−18.1530
	−6.5	9.4957	2.908E−2	1.6020	1.2866	−9.3181	−9.6335	−2.9685	−13.8886
	−5.5	8.4957	2.551E−1	3.7737	3.4583	−5.9964	−6.3118	−0.7968	−10.5669
	−4.5	7.4957	8.821E−1	5.0143	4.6989	−3.6058	−3.9212	0.4438	−8.1763
	−3.5	6.4957	1.212	5.3323	5.0169	−2.1378	−2.4532	0.7618	−6.7083
	−2.5	5.4957	6.640E−1	4.7303	4.4148	−1.5898	−1.9053	0.1597	−6.1604
	−1.5	4.4957	1.442E−1	3.2035	2.8880	−1.9666	−2.2821	−1.3670	−6.5371
	−0.5	3.4957	1.231E−2	0.7422	0.4268	−3.2779	−3.5933	−3.8283	−7.8484
40	−6.5	10.1889	3.256E−3	−0.4558	−0.7712	−12.1730	−12.4884	−5.0263	−16.7435
	−5.5	9.1889	5.484E−2	2.3681	2.0526	−8.1991	−8.5146	−2.2024	−12.7697
	−4.5	8.1889	3.611E−1	4.2529	3.9375	−5.1643	−5.4797	−0.3176	−9.7348
	−3.5	7.1889	9.402E−1	5.2098	4.8943	−3.0574	−3.3729	0.6392	−7.6280
	−2.5	6.1889	9.744E−1	5.2455	4.9301	−1.8717	−2.1871	0.6750	−6.4422
	−1.5	5.1889	4.021E−1	4.3605	4.0450	−1.6067	−1.9222	−0.2101	−6.1773
	−0.5	4.1889	6.566E−2	2.5482	2.2327	−2.2690	−2.5845	−2.0224	−6.8396
	0.5	3.1889	4.196E−3	−0.2022	−0.5177	−3.8694	−4.1849	−4.7728	−8.4400
80	−5.5	9.8820	7.508E−3	0.5113	0.1959	−10.8530	−11.1684	−4.0592	−15.4235
	−4.5	8.8820	9.467E−2	3.0458	2.7303	−7.1686	−7.4840	−1.5248	−11.7391
	−3.5	7.8820	4.684E−1	4.6448	4.3293	−4.4195	−4.7350	0.0742	−8.9901
	−2.5	6.8820	9.189E−1	5.3185	5.0031	−2.5958	−2.9113	0.7480	−7.1663
	−1.5	5.8820	7.181E−1	5.0720	4.7566	−1.6923	−2.0077	0.5015	−6.2628
	−0.5	4.8820	2.232E−1	3.9034	3.5880	−1.7109	−2.0264	−0.6671	−6.2815
	0.5	3.8820	2.736E−2	1.8045	1.4891	−2.6598	−2.9753	−2.7660	−7.2303
	1.5	2.8820	1.308E−3	−1.2362	−1.5516	−4.5505	−4.8659	−5.8067	−9.1210

Table 4 continued

W	$\log w$	$\log(W/w)$	$w\tilde{N}_{st}\,\mathrm{d}\log w$	$\log(\Omega_r w\tilde{N}_{st}\mathrm{d}\log w)$	$\log(\Omega_t w\tilde{N}_{st}\mathrm{d}\log w)$	$\log(\Omega_{1r} w\tilde{N}_{st}\mathrm{d}\log w)$	$\log(\Omega_{1t} w\tilde{N}_{st}\mathrm{d}\log w)$	$\log(\Omega_2 w\tilde{N}_{st}\mathrm{d}\log w)$	$\log(\Omega_3 w\tilde{N}_{st}\mathrm{d}\log w)$
160	−5.5	10.5752	6.520E−4	−1.8006	−2.1161	−13.9621	−14.2775	−6.3712	−18.5326
	−4.5	9.5752	1.583E−2	1.3892	1.0738	−9.6222	−9.9377	−3.1813	−14.1928
	−3.5	8.5752	1.497E−1	3.6354	3.3199	−6.2261	−6.5415	−0.9352	−10.7966
	−2.5	7.5752	5.570E−1	4.9496	4.6341	−3.7619	−4.0773	0.3790	−8.3324
	−1.5	6.5752	8.236E−1	5.3407	5.0253	−2.2207	−2.5362	0.7702	−6.7913
	−0.5	5.5752	4.853E−1	4.8118	4.4963	−1.5997	−1.9151	0.2413	−6.1702
	0.5	4.5752	1.135E−1	3.3588	3.0433	−1.9026	−2.2181	−1.2117	−6.4732
	1.5	3.5752	1.043E−2	0.9722	0.6568	−3.1392	−3.4547	−3.5983	−7.7098
320	−4.5	10.2683	1.683E−3	−0.7207	−1.0362	−12.5293	−12.8447	−5.2913	−17.0998
	−3.5	9.2683	3.055E−2	2.1783	1.8628	−8.4803	−8.7958	−2.3923	−13.0508
	−2.5	8.2683	2.167E−1	4.1372	3.8218	−5.3713	−5.6868	−0.4333	−9.9419
	−1.5	7.2683	6.071E−1	5.1675	4.8520	−3.1911	−3.5065	0.5969	−7.7616
	−0.5	6.2683	6.769E−1	5.2763	4.9609	−1.9323	−2.2477	0.7058	−6.5028
	0.5	5.2683	3.006E−1	4.4646	4.1491	−1.5940	−1.9095	−0.1060	−6.1645
	1.5	4.2683	5.285E−2	2.7263	2.4108	−2.1823	−2.4977	−1.8443	−6.7528
	2.5	3.2683	3.640E−3	0.0509	−0.2646	−3.7077	−4.0232	−4.5197	−8.2783
640	−3.5	9.9615	3.971E−3	0.2695	−0.0459	−11.1861	−11.5016	−4.3010	−15.7567
	−2.5	8.9615	5.396E−2	2.8788	2.5633	−7.4269	−7.7424	−1.6918	−11.9974
	−1.5	7.9615	2.875E−1	4.5517	4.2362	−4.6040	−4.9194	−0.0188	−9.1745
	−0.5	6.9615	6.068E−1	5.2987	4.9832	−2.7070	−3.0225	0.7281	−7.2775
	0.5	5.9615	5.102E−1	5.1253	4.8098	−1.7304	−2.0459	0.5547	−6.3010
	1.5	4.9615	1.707E−1	4.0301	3.7147	−1.6755	−1.9910	−0.5404	−6.2461
	2.5	3.9615	2.254E−2	2.0055	1.6901	−2.5501	−2.8656	−2.5650	−7.1207
	3.5	2.9615	1.161E−3	−0.9599	−1.2753	−4.3656	−4.6810	−5.5304	−8.9361

Example 3

In table 5 are shown stomach data for four *Pr* sizes from an environment with several *pr* species. Data are given for two *pr* species with the specifications: The environmental weight distributions are log normal (μ, τ^2) and the parameters are:

	μ	τ^2	Relative numbers
*pr*1	−5	1	7503
*pr*2	−3.5	1	3488

It is further known that

$$\phi = N_2 C_2 W^{1-\nu}.$$

Determination of ν: (iii-*b*) gives: $w\tilde{N}_{st}\,\mathrm{d}\log w/(wN\mathrm{d}\log w)/stctt =$

$$\frac{\rho W^{\nu-1}}{N_2 C_2\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)} \exp(-(\log u - \eta)^2/2/\sigma^2).$$

This means that we for each W and pr can produce a parabola by using the factor

$1/(wN\mathrm{dlog}w)/stctt$

where $wN\mathrm{dlog}w$ is found by means of μ, τ^2, and relative numbers and we get

$$\psi(\log u) = \log\left\{\frac{\rho}{N_2C_2\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} + (\nu - 1)\log W - \frac{(\log u - \eta)^2}{2\sigma^2}$$

*pr*1
$$\psi(\log u) = -25.1642 + 5.6636\log u - 0.4059\log^2 u = -5.4079 - (\log u - 6.9766)^2/2/1.2318. \ (10\ \mathrm{g}).$$
$$\psi(\log u) = -24.8793 + 5.5666\log u - 0.4045\log^2 u = -5.7279 - (\log u - 6.8808)^2/2/1.2361 \ (40\ \mathrm{g}).$$
$$\psi(\log u) = -24.0415 + 5.3435\log u - 0.3963\log^2 u = -6.0293 - (\log u - 6.7417)^2/2/1.2617. \ (160\ \mathrm{g}).$$
$$\psi(\log u) = -23.3879 + 5.1788\log u - 0.3924\log^2 u = -6.3008 - (\log u - 6.5989)^2/2/1.2742. \ (640\ \mathrm{g}).$$

*pr*2
$$\psi(\log u) = -12.8815 + 2.5242\log u - 0.2098\log^2 u = -5.2890 - (\log u - 6.0157)^2/2/2.3832. \ (10\ \mathrm{g}).$$
$$\psi(\log u) = -12.8512 + 2.4523\log u - 0.2075\log^2 u = -5.6057 - (\log u - 5.9092)^2/2/2.4096. \ (40\ \mathrm{g}).$$
$$\psi(\log u) = -12.9568 + 2.4238\log u - 0.2086\log^2 u = -5.9160 - (\log u - 5.8097)^2/2/2.3969. \ (160\ \mathrm{g}).$$
$$\psi(\log u) = -12.7601 + 2.3168\log u - 0.2048\log^2 u = -6.2079 - (\log u - 5.6562)^2/2/2.4414. \ (640\ \mathrm{g}).$$

Within each pr group the parabolas should only differ in the constant term, and the actual differences are due to the coarse grouping of the data. The two sets of constant terms

−25.1642, −24.8793, −24.0415, −23.3879
−5.4079, −5.7279, −6.0293, −6.3008

should both conform to a linear function in $\log W$ with slope $\nu-1$ and linear regression give

$$\log(\rho/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(\kappa t_l + \kappa t_0))) - \eta^2/2/\sigma^2 = -26.3175$$
$$\nu - 1 = 0.4448. \quad \nu = 1.4448. \ \text{(First set)}.$$
$$\log(\rho/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(\kappa t_l + \kappa t_0))) = -4.9245$$
$$\nu - 1 = -0.2150. \quad \nu = 0.7850. \ \text{(Second set)}.$$

The results for $pr2$ are

$$\log(\rho/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(\kappa t_l + \kappa t_0))) + \eta^2/2/\sigma^2 = -12.9441.$$
$$\nu-1 = 0.0187. \quad \nu = 1.0187. \ \text{(First set)}.$$
$$\log(\rho/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(\kappa t_l + \kappa t_0) = -4.7852.$$
$$\nu-1 = -0.2212. \quad \nu = 0.7788. \ \text{(Second set)}.$$

It can be shown that the figures found from first sets are much more influenced by the biases induced by the grouping than the figures found from second sets. This is in agreement with the $\nu = 0.7778$ used for constructing table 5 and $\nu = 0.7778$ will be used from now on in this part of example 3.

Determination of σ^2 and η:

By using the factor

$$W^{1-\nu}/(w N \mathrm{dlog} w)/stctt = W^{0.2222}/(w N \mathrm{dlog} w)/stctt$$

we get single parabolas for each *pr* species:

$$\psi(\log u) = \log\left\{\frac{\rho}{N_2 C_2\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2}{2\sigma^2} + \frac{\eta}{\sigma^2}\log u - \frac{\log^2 u}{2\sigma^2}$$

pr1

$$\psi(\log u) = -25.5021 + 5.9792\log u - 0.4338\log^2 u$$
$$\sigma^2_{pr1} = 1.1526. \quad \eta_{pr1} = 6.8917.$$

pr2

$$\psi(\log u) = -12.6003 + 2.6449\log u - 0.2235\log^2 u$$
$$\sigma^2_{pr2} = 2.2371. \quad \eta_{pr2} = 5.9170.$$

The values can be compared with the values found when determining ν.

Determination of ρ:

We choose $\rho_{pr2} = 1$ and $N_2 = 1$ and the ψ intercept is

$$\text{intercept} = \log\left\{\frac{\rho}{1 C_2\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2}{2\sigma^2}$$

or

$$C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(\kappa t_l + \kappa t_0) = 118.55$$

and $\rho_{pr1} = 0.8839.$

The ν determination gives

$$\rho_{pr1} = \exp(-4.9245 + 4.7852) = 0.8700.$$

Determination of $\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$, κ_1, $q\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))/(N_2C_2)$, and r:

(vi-*b*) gives us the equation

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))}\left(W^{-m+\kappa_1} + \frac{q}{N_2 C_2} W^{\nu-1-m+\kappa_1+1}\right) = \frac{1}{stctt}$$

and the four W_{P_r} values give the solutions

$$\kappa_0/h/(\kappa t_l+1-\exp(-\kappa t_{00})) = 49.048.$$
$$-m+\kappa_1 = -0.8097.\ \ \kappa_1 = -0.2497.$$
$$q\kappa_0/h/(\kappa t_l+1-\exp(-\kappa t_{00}))/(N_2C_2) = 246.07$$
$$\nu-1-m+\kappa_1+r = -1.1819.\ \ r = -0.1500.$$

Determination of f:

$$f_w = \frac{\kappa_0}{h(\kappa t_l+1-\exp(-\kappa t_{00}))}\Bigg/\left(\frac{\kappa_0}{h(\kappa t_l+1-\exp(-\kappa t_{00}))}\left(W^{1-\nu}+\frac{q}{N_2C_2}W^r\right)\right)$$

$$f_{10} = 0.3196 \quad f_{40} = 0.4403 \quad f_{160} = 0.5686 \quad f_{640} = 0.6883.$$

The calculations have only been made for *stctt* but could of course be made for *stctr* just as well.

Table 5. Example 3. First environment. Stomach data.

$d\log w = 1.$

stctt(10) = 4.2035E−2. *stctt*(40) = 1.7796E−1. *stctt*(160) = 7.0606E−1. *stctt*(640) = 2.6260.

$\Omega_1 = 1/(wN d\log w)/stctt.\ \ \Omega_2 = W^{0.2222}/(wN d\log w)/stctt.$

*pr*1

W	log$w$	log(W/w)	wN dlog$w$	$w\tilde{N}_{st}$ dlog$w$	log $\Omega_1 w\tilde{N}_{st}$ dlog$w$	log $\Omega_2 w\tilde{N}_{st}$ dlog$w$
10	−7.5	9.8026	1.606E 2	1.169E−3	−8.6609	−8.1492
	−6.5	8.8026	1.020E 3	5.056E−2	−6.7425	−6.2309
	−5.5	7.8026	2.561E 3	3.635E−1	−5.6909	−5.1793
	−4.5	6.8026	2.561E 3	4.696E−1	−5.4348	−4.9231
	−3.5	5.8026	1.020E 3	1.105E−1	−5.9604	−5.4487
	−2.5	4.8026	1.606E 2	4.439E−3	−7.3266	−6.8149
40	−6.5	10.1889	1.020E 3	7.050E−3	−10.1558	−9.3361
	−5.5	9.1889	2.561E 3	1.742E−1	−7.8697	−7.0500
	−4.5	8.1889	2.561E 3	7.345E−1	−6.4305	−5.6108
	−3.5	7.1889	1.020E 3	5.644E−1	−5.7730	−4.9532
	−2.5	6.1889	1.606E 2	7.793E−2	−5.9044	−5.0847
	−1.5	5.1889	9.890	1.788E−3	−6.8920	−6.0723
160	−5.5	10.5752	2.561E 3	1.280E−2	−11.8585	−10.7307
	−4.5	9.5752	2.561E 3	1.821E−1	−9.2033	−8.0755
	−3.5	8.5752	1.020E 3	4.536E−1	−7.3697	−6.2419
	−2.5	7.5752	1.606E 2	2.070E−1	−6.3056	−5.1778
	−1.5	6.5752	9.890	1.663E−2	−6.0398	−4.9120
640	−4.5	10.9615	2.561E 3	7.014E−3	−13.7734	−12.3375
	−3.5	9.9615	1.020E 3	5.796E−2	−10.7406	−9.3047
	−2.5	8.9615	1.606E 2	8.568E−2	−8.5013	−7.0654
	−1.5	7.9615	9.890	2.313E−2	−7.0238	−5.5879
	−0.5	6.9615	2.355E−1	1.072E−3	−6.3572	−4.9213

Table 5 continued

*pr*2

W	$\log w$	$\log(W/w)$	$WN\ d\log w$	$w\tilde{N}_{st}\ d\log w$	$\log \Omega_1 w\tilde{N}_{st}\ d\log w$	$\log \Omega_2 w\tilde{N}_{st}\ d\log w$
	−6.5	8.8026	2.085E 1	8.622E−4	−6.9240	−6.4123
	−5.5	7.8026	2.114E 2	2.308E−2	−5.9533	−5.4416
	−4.5	6.8026	8.432E 2	1.564E−1	−5.4235	−4.9118
10	−3.5	5.8026	1.336E 3	2.779E−1	−5.3085	−4.7968
	−2.5	4.8026	8.432E 2	1.313E−1	−5.5982	−5.0865
	−1.5	3.8026	2.114E 2	1.621E−2	−6.3066	−5.7949
	−0.5	2.8026	2.085E 1	5.040E−4	−7.4609	−6.9493
	−5.5	9.1889	2.114E 2	1.486E−2	−7.8363	−7.0165
	−4.5	8.1889	8.432E 2	1.891E−1	−6.6766	−5.8568
40	−3.5	7.1889	1.336E 3	6.215E−1	−5.9465	−5.1268
	−2.5	6.1889	8.432E 2	5.418E−1	−5.6238	−4.8040
	−1.5	5.1889	2.114E 2	1.248E−1	−5.7081	−4.8883
	−0.5	4.1889	2.085E 1	7.377E−3	−6.2204	−5.4007
	−5.5	10.5752	2.114E 2	3.512E−3	−10.6571	−9.5292
	−4.5	9.5752	8.432E 2	8.458E−2	−8.8591	−7.7313
	−3.5	8.5752	1.336E 3	5.173E−1	−7.5082	−6.3804
160	−2.5	7.5752	8.432E 2	8.314E−1	−6.5737	−5.4459
	−1.5	6.5752	2.114E 2	3.552E−1	−6.0406	−4.9128
	−0.5	5.5752	2.085E 1	3.955E−2	−5.9194	−4.7916
	0.5	4.5752	7.996E−1	1.106E−3	−6.2352	−5.1074
	−4.5	10.9615	8.432E 2	1.395E−2	−11.9749	−10.5390
	−3.5	9.9615	1.336E 3	1.600E−1	−9.9955	−8.5596
640	−2.5	8.9615	8.432E 2	4.752E−1	−8.4467	−7.0108
	−1.5	7.9615	2.114E 2	3.747E−1	−7.3007	−5.8648
	−0.5	6.9615	2.085E 1	7.797E−2	−6.5541	−5.1182
	0.5	5.9615	7.996E−1	4.149E−3	−6.2266	−4.7907

Table 6 shows stomach data from another similar environment with several *pr* species. *pr*1 and *pr*2 have the same specifications as in the first environment, but here it is known that $\nu + r = 1$.

Determination of κ_1:

$\nu + r = 1$ means that f is independent of W_{Pr} and this fact gives

$$\log(stctt) = \log\left(\frac{fh(\kappa t_l\ + 1\ 4\ \exp(-\kappa t_{00}))}{\kappa_0}\right) + (m - \kappa_1)\log W$$

and we find using linear regression

$m - \kappa_1 = 0.8100.\quad \kappa_1 = -0.2500.$

$\log(\kappa_0/(fh(\kappa t_l + 1 - \exp(-\kappa t_{00})))) = 4.1189.$

$\kappa_0/(fh(\kappa t_l + 1 - \exp(-\kappa t_{00}))) = 61.492.$

Determination of ν:

(iii-b) gives

$$\log\left(\frac{w\widetilde{N}_{st}\,\mathrm{dlog}w}{wN\,\mathrm{dlog}w\ \ stctt}\right) = \log\left\{\frac{\rho}{N_2C_2\left(\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\}$$

$$+ (\nu - 1)\log W - \frac{(\log u - \eta)^2}{2\sigma^2}$$

and

$$\log\left(\frac{w\widetilde{N}_{st}\,\mathrm{dlog}w}{wN\,\mathrm{dlog}w}\right) = \log\left(\frac{\rho h(\kappa t_l + \kappa t_0)}{\kappa_0(N_2C_2 + q)}\right) + (\nu - 1 + m - \kappa_1)\log W - \frac{(\log u - \eta)^2}{2\sigma^2}\,.$$

The data thus give us 2×2 sets of parabolas:

*pr*1
$\psi(\log u) = -27.1129 + 5.6028\log u - 0.4012\log^2 u$
$= -7.5520 - (\log u - 6.9826)^2/2/1.2463.$ (10 g).
$\psi(\log u) = -26.5020 + 5.5666\log u - 0.4045\log^2 u$
$= -7.3506 - (\log u - 6.8808)^2/2/1.2361.$ (40 g).
$\psi(\log u) = -25.1482 + 5.3435\log u - 0.3963\log^2 u$
$= -7.1360 - (\log u - 6.7417)^2/2/1.2617.$ (160 g).
$\psi(\log u) = -23.9786 + 5.1788\log u - 0.3924\log^2 u$
$= -6.8915 - (\log u - 6.5989)^2/2/1.2742.$ (640 g).

*pr*2
$\psi(\log u) = -14.8859 + 2.4766\log u - 0.2058\log^2 u$
$= -7.4350 - (\log u - 6.0170)^2/2/2.4295.$ (10 g).
$\psi(\log u) = -14.4739 + 2.4523\log u - 0.2075\log^2 u$
$= -7.2284 - (\log u - 5.9092)^2/2/2.4096.$ (40 g).
$\psi(\log u) = -14.0635 + 2.4238\log u - 0.2086\log^2 u$
$= -7.0126 - (\log u - 5.8097)^2/2/2.3969.$ (160 g).
$\psi(\log u) = -13.3507 + 2.3168\log u - 0.2048\log^2 u$
$= -6.7985 - (\log u - 5.6562)^2/2/2.4414.$ (640 g).

and

*pr*1
$\psi(\log u) = -29.3669 + 5.6028\log u - 0.4012\log^2 u$
$= -9.8060 - (\log u - 6.9825)^2/2/1.2463.$ (10 g).
$\psi(\log u) = -27.6330 + 5.5666\log u - 0.4045\log^2 u$
$= -8.4816 - (\log u - 6.8808)^2/2/1.2361.$ (40 g).
$\psi(\log u) = -25.1563 + 5.3435\log u - 0.3963\log^2 u$
$= -7.1441 - (\log u - 6.7417)^2/2/1.2617.$ (160 g).
$\psi(\log u) = -22.8638 + 5.1788\log u - 0.3924\log^2 u$
$= -5.7767 - (\log u - 6.5989)^2/2/1.2742.$ (640 g).

*pr*2 $\psi(\log u) = -17.1399 + 2.4766\log u - 0.2058\log^2 u$
$= -9.6890 - (\log u - 6.0170)^2/2/2.4295.$ (10 g).
$\psi(\log u) = -15.6049 + 2.4523\log u - 0.2075\log^2 u$
$= -8.3594 - (\log u - 5.9092)^2/2/2.4096.$ (40 g).
$\psi(\log u) = -14.0716 + 2.4238\log u - 0.2086\log^2 u$
$= -7.0308 - (\log u - 5.8097)^2/2/2.3969.$ (160 g).
$\psi(\log u) = -12.2360 + 2.3168\log u - 0.2048\log^2 u$
$= -5.6838 - (\log u - 5.6562)^2/2/2.4414.$ (640 g).

Linear regressions on constant terms give

*pr*1 $\log(\rho(\kappa t_l + \kappa t_0)/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00})))) - \eta^2/2/\sigma^2 = -29.0856$
$\nu - 1 = 0.7759.$ $\nu = 1.7759.$ (First set).
$\log(\rho(\kappa t_l + \kappa t_0)/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00})))) = -7.9267$
$\nu - 1 = 0.1584.$ $\nu = 1.1584.$ (Second set).

*pr*2 $\log(\rho(\kappa t_l + \kappa t_0)/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00})))) - \eta^2/2/\sigma^2 = -15.7790$
$\nu - 1 = 0.3618.$ $\nu = 1.3618.$ (First set).
$\log(\rho(\kappa t_l + \kappa t_0)/(N_2C_2(\kappa t_l + 1 - \exp(-\kappa t_{00})))) = -7.7904$
$\nu - 1 = -0.1533.$ $\nu = 1.1533.$ (Second set).

*pr*1 $\log(\rho h(\kappa t_l + \kappa t_0)/\kappa_0/(N_2C_2 + q)) - \eta^2/2/\sigma^2 = -33.2047$
$\nu - 1 + m - \kappa_1 = 1.5859.$ $\nu = 1.7759.$ (First set).
$\log(\rho h(\kappa t_l + \kappa t_0)/\kappa_0/(N_2C_2 + q)) = -12.0458$
$\nu - 1 + m - \kappa_1 = 0.9684.$ $\nu = 1.1584.$ (Second set).

*pr*2 $\log(\rho h(\kappa t_l + \kappa t_0)/\kappa_0/(N_2C_2 + q)) - \eta^2/2/\sigma^2 = -19.8981$
$\nu - 1 + m - \kappa_1 = 1.1718.$ $\nu = 1.3618.$ (First set).
$\log(\rho h(\kappa t_l + \kappa t_0)/\kappa_0/(N_2C_2 + q) = -11.9088$
$\nu - 1 + m - \kappa_1 = 0.9626.$ $\nu = 1.1526.$ (Second set).

The ν used for constructing table 6 was 1.15 and we again see that figures based on second sets are much less influenced by the grouping than those based on first sets. $\nu = 1.15$ will be used in the rest of this example.

Determination of σ^2 and η:

Common parabolas are produced by means of the weighting factors

$\Omega_3 = W^{1-\nu}/(wN\mathrm{d}\log w)/stctt$

and

$\Omega_4 = W^{1-\nu-m+\kappa_1}/(wN\mathrm{d}\log w)$

giving

$$\psi(\log u) = \log\left\{\frac{\rho}{N_2C_2\left(\dfrac{\kappa t_l + 1 + \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\right\} - \frac{\eta^2}{2\sigma^2} + \frac{\eta}{\sigma^2}\log u - \frac{\log^2 u}{2\sigma^2}$$

$$\psi(\log u) = \log\left\{\frac{\rho}{\dfrac{\kappa_0}{h(\kappa t_l + \kappa t_0)}(N_2C_2 + q)}\right\} - \frac{\eta^2}{2\sigma^2} + \frac{\eta}{\sigma^2}\log u - \frac{\log^2 u}{2\sigma^2}$$

$pr1$ $\psi(\log u) = -28.4942 + 5.9869\log u - 0.4351\log^2 u$
$\sigma^2_{pr1} = 1.1492.$ $\eta_{pr1} = 6.8799.$ (Ω_3).
$\psi(\log u) = -32.6132 + 5.9869\log u - 0.4351\log^2 u.$ (Ω_4).
σ^2 and η as for Ω_3.

$pr2$ $\psi(\log u) = -15.5117 + 2.6219\log u - 0.2222\log^2 u.$
$\sigma^2_{pr2} = 2.2502.$ $\eta_{pr2} = 5.8999.$ (Ω_3).
$\psi(\log u) = -19.6307 + 2.6219\log u - 0.2222\log^2 u$ (Ω_4).
σ^2 and η as for Ω_3.

The found values can again be compared with the values from the ν determination.

Determination of ρ:

The last parabola intercepts give

$$\rho_{pr1}/\rho_{pr2} = \exp(-28.4942 + 5.9869/4/0.4351 + 15.5117 - 2.6219/4/0.2222)$$
$$= 0.8849. \quad (\Omega_3).$$
$$= 0.8849. \quad (\Omega_4).$$

The ν determination gives

$$\rho_{pr1}/\rho_{pr2} = \exp(-7.9267 + 7.7904) = 0.8726.$$
$$\rho_{pr1}/\rho_{pr2} = \exp(-12.0458 + 11.9088) = 0.8720.$$

As any choice of κ_0 and q that keeps

$$\kappa_0/f = \kappa_0(1 + q/(N_2C_2))$$

constant gives the same stomach data we are unable to do $\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$ q/N_2C_2 and f determinations.

Table 6. Example 3, second experiment, $\nu + r = 1$. Stomach data.

$d\log w = 1.$

$stctt(10) = 1.0499E-1.$ $stctt(40) = 3.2270E-1.$ $stctt(160) = 9.9190E-1.$ $stctt(640) = 3.0488.$

$\Omega_1 = 1/(wN d\log w)/stctt.$ $\Omega_2 = 1/stctt.$ $\Omega_3 = W^{1-\nu}/(wN d\log w)/stctt.$ $\Omega_4 = W^{1-\nu-m+\kappa_1}/(wN d\log w)$

$pr1$

W	log$w$	log(W/w)	wNdlog$w$	$w\bar{N}_{st}$ dlog$w$	log$(\Omega_1 w\bar{N}_{st}$dlog$w)$	log$(\Omega_2 w\bar{N}_{st}$dlog$w)$	log$(\Omega_3 w\bar{N}_{st}$dlog$w)$	log$(\Omega_4 w\bar{N}_{st}$dlog$w)$
	−6.5	8.8026	1.020E 3	1.488E−2	−8.8812	−11.1351	−9.2266	−13.3456
	−5.5	7.8026	2.561E 3	1.070E−1	−7.8296	−10.0835	−8.1750	−12.2940
10	−4.5	6.8026	2.561E 3	1.382E−1	−7.5734	−9.8273	−7.9188	−12.0378
	−3.5	5.8026	1.020E 3	3.252E−2	−8.0991	−10.3530	−8.4445	−12.5635
	−2.5	4.8026	1.606E 2	1.306E−3	−9.4653	−11.7192	−9.8107	−13.9297

Table 6 continued

W	log$w$	$\log(W/w)$	$wN\,\mathrm{d}\log w$	$w\tilde{N}_{st}\,\mathrm{d}\log w$	$\log(\Omega_1 w\tilde{N}_{st}\,\mathrm{d}\log w)$	$\log(\Omega_2 w\tilde{N}_{st}\,\mathrm{d}\log w)$	$\log(\Omega_3 w\tilde{N}_{st}\,\mathrm{d}\log w)$	$\log(\Omega_4 w\tilde{N}_{st}\,\mathrm{d}\log w)$
40	−6.5	10.1889	1.020E 3	2.523E−3	−11.7785	−12.9095	−12.3318	−16.4508
	−5.5	9.1889	2.561E 3	6.233E−2	−9.4924	−10.6234	−10.0457	−14.1648
	−4.5	8.1889	2.561E 3	2.629E−1	−8.0532	−9.1842	−8.6065	−12.7255
	−3.5	7.1889	1.020E 3	2.020E−1	−7.3956	−8.5267	−7.9490	−12.0680
	−2.5	6.1889	1.606E 2	2.789E−2	−7.5271	−8.6581	−8.0804	−12.1994
	−1.5	5.1889	9.890	6.399E−4	−8.5147	−9.6457	−9.0680	−13.1871
160	−5.5	10.5752	2.561E 3	5.945E−3	−12.9652	−12.9733	−13.7265	−17.8455
	−4.5	9.5752	2.561E 3	8.459E−2	−10.3099	−10.3181	−11.0712	−15.1902
	−3.5	8.5752	1.020E 3	2.107E−1	−8.4764	−8.4845	−9.2376	−13.3567
	−2.5	7.5752	1.606E 2	9.616E−2	−7.4123	−7.4204	−8.1735	−12.2926
	−1.5	6.5752	9.890	7.727E−3	−7.1465	−7.1546	−7.9078	−12.0268
640	−4.5	10.9615	2.561E 3	4.511E−3	−14.3641	−13.2493	−15.3333	−19.4523
	−3.5	9.9615	1.020E 3	3.728E−2	−11.3313	−10.2165	−12.3005	−16.4195
	−2.5	8.9615	1.606E 2	5.511E−2	−9.0919	−7.9771	−10.0611	−14.1801
	−1.5	7.9615	9.890	1.487E−2	−7.6144	−6.4997	−8.5837	−12.7027
	−0.5	6.9615	2.355E−1	6.897E−4	−6.9478	−5.8330	−7.9170	−12.0361
pr2								
10	−5.5	7.8026	2.114E 2	6.790E−3	−8.0920	−10.3459	−8.4374	−12.5564
	−4.5	6.8026	8.432E 2	4.601E−2	−7.5621	−9.8160	−7.9075	−12.0265
	−3.5	5.8026	1.336E 3	8.177E−2	−7.4471	−9.7010	−7.7925	−11.9115
	−2.5	4.8026	8.432E 2	3.864E−2	−7.7368	−9.9907	−8.0822	−12.2012
	−1.5	3.8026	2.114E 2	4.769E−3	−8.4452	−10.6992	−8.7906	−12.9097
40	−5.5	9.1889	2.114E 2	5.319E−3	−9.4590	−10.5900	−10.0123	−14.1313
	−4.5	8.1889	8.432E 2	6.767E−2	−8.2992	−9.4303	−8.8526	−12.9716
	−3.5	7.1889	1.336E 3	2.224E−1	−7.5692	−8.7002	−8.1225	−12.2416
	−2.5	6.1889	8.432E 2	1.939E−1	−7.2464	−8.3775	−7.7998	−11.9188
	−1.5	5.1889	2.114E 2	4.468E−2	−7.3307	−8.4618	−7.8841	−12.0031
	−0.5	4.1889	2.085E 1	2.640E−3	−7.8431	−8.9741	−8.3964	−12.5155
160	−5.5	10.5752	2.114E 2	1.632E−3	−11.7637	−11.7719	−12.5250	−16.6440
	−4.5	9.5752	8.432E 2	3.929E−2	−9.9658	−9.9739	−10.7270	−14.8461
	−3.5	8.5752	1.336E 3	2.403E−1	−8.6149	−8.6230	−9.3762	−13.4952
	−2.5	7.5752	8.432E 2	3.862E−1	−7.6804	−7.6885	−8.4416	−12.5607
	−1.5	6.5752	2.114E 2	1.650E−1	−7.1473	−7.1554	−7.9086	−12.0276
	−0.5	5.5752	2.085E 1	1.837E−2	−7.0261	−7.0342	−7.7874	−11.9064
	0.5	4.5752	7.996E−1	5.138E−4	−7.3419	−7.3500	−8.1032	−12.2222
640	−4.5	10.9615	8.432E 2	8.972E−3	−12.5655	−11.4508	−13.5348	−17.6538
	−3.5	9.9615	1.336E 3	1.029E−1	−10.5862	−9.4714	−11.5554	−15.6744
	−2.5	8.9615	8.432E 2	3.056E−1	−9.0373	−7.9226	−10.0066	−14.1256
	−1.5	7.9615	2.114E 2	2.410E−1	−7.8913	−6.7766	−8.8606	−12.9796
	−0.5	6.9615	2.085E 1	5.015E−2	−7.1447	−6.0300	−8.1140	−12.2330
	0.5	5.9615	7.996E−1	2.669E−3	−6.8173	−5.7025	−7.7865	−11.9055

Example 4

A predator *Pr* preys on six *pr* species. The *pr* species have log normal environmental weight distributions (μ, τ^2) with the specifications

	μ	τ^2	Relative number $= N^*$
*pr*1	−5	1	8000
*pr*2	−3.5	1	4000
*pr*3	−2	1	3000
*pr*4	1.5	1	250
*pr*5	−4	1	3500
*pr*6	−3	1	2500

The situation is similar to example 3 but in this case no simple expression for ψ is available.

Table 7 gives stomach data for four *Pr* sizes.

Determination of σ^2 *and* η:

We start by determining the ingestion weight distribution from (iii-*b*)

$$\widetilde{N}_{st}\,dw\,\frac{\kappa_0 W^{\kappa_1}}{\kappa t_l + \kappa t_0} = \frac{\rho \exp(-(\log(W/w)-\eta)^2/2/\sigma^2)}{\phi + qW^r}$$

$$* \;\frac{N^*}{\sqrt{2\pi}\,\tau\, w}\exp(-(\log w - \mu)^2/2/\tau^2)\,dw\,h\,W^m$$

or

$$\widetilde{N}_{st} = \frac{\rho N^* h W^m \sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}{\dfrac{\kappa_0 W^{\kappa_1}}{\kappa t_l + \kappa t_0}(\phi + qW^r)\tau}\exp\left\{\frac{\left(\dfrac{\mu\sigma^2-(\eta-\log W)\tau^2}{\sigma^2+\tau^2}\right)^2}{2\left(\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}\right)} - \frac{(\log W-\eta)^2}{2\sigma^2} - \frac{\mu^2}{2\tau^2}\right\}$$

$$* \;\frac{1}{w\sqrt{2\pi}\sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}\exp\left\{\frac{\left(\log w - \dfrac{\mu\sigma^2-(\eta-\log W)\tau^2}{\sigma^2+\tau^2}\right)^2}{2\left(\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}\right)}\right\} \qquad \text{(vii)}$$

and this means that $\widetilde{N}_{st}$ is log normally shaped with the following parameters stomach mean log ingestion weight

$$\hat{\mu} = \frac{\mu\sigma^2 - (\eta - \log W)\tau^2}{\sigma^2+\tau^2} \qquad \text{(viii)}$$

stomach variance of log ingestion weight

$$\hat{\tau}^2 = \frac{\sigma^2\tau^2}{\sigma^2+\tau^2} \qquad \text{(ix)}$$

total recognizable number in stomach

$$\widetilde{N}_{st}^{*} = \frac{\rho N^{*} h W^{m} \sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}{\dfrac{\kappa_0 W^{\kappa_1}}{\kappa t_l + \kappa t_0}(\phi + qW^{r})\tau} \exp\left\{\frac{\left(\dfrac{\mu\sigma^2-(\eta-\log W)\tau^2}{\sigma^2+\tau^2}\right)^2}{2\left(\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}\right)} - \frac{(\log W-\eta)^2}{2\sigma^2} - \frac{\mu^2}{2\tau^2}\right\}$$

$$= \frac{\rho N^{*} h W^{m} \sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}}{\dfrac{\kappa_0 W^{\kappa_1}}{\kappa t_l + \kappa t_0}(\phi + qW^{r})\tau} \exp\left(-\frac{1}{2(\sigma^2+\tau^2)}(\mu+\eta-\log W)^2\right). \qquad \text{(x)}$$

From (viii) and (ix) we now get

$$\eta = \frac{\mu\hat{\tau}^2 - \hat{\mu}\tau^2}{\tau^2 - \hat{\tau}^2} + \log W \qquad \text{(xi)}$$

and

$$\sigma^2 = \frac{\tau^2\hat{\tau}^2}{\tau^2 - \hat{\tau}^2}. \qquad \text{(xii)}$$

Table 7 gives using (xi) and (xii):

pr	W	η	σ^2	*pr*	W	η	σ^2
	10	7.0000	1.0000		10	4.0000	0.4000
*pr*1	40	7.0001	1.0000	*pr*4	40	4.0000	0.4000
	160	7.0000	1.0000		160	4.0001	0.4000
	640	7.0001	1.0000		640	4.0001	0.4000
	10	6.0000	2.0000		10	6.5000	2.5001
*pr*2	40	6.0000	2.0000	*pr*5	40	6.4999	2.5001
	160	5.9999	2.0000		160	6.4998	2.5001
	640	5.9999	2.0000		640	6.4998	2.5001
	10	5.0001	0.8000		10	4.5001	1.5000
*pr*3	40	5.0000	0.8000	*pr*6	40	4.5001	1.5000
	160	5.0000	0.8000		160	4.5002	1.5000
	640	4.9999	0.8000		640	4.5002	1.5000

Determination of ρ:

(x) can be written as

$$\frac{\rho h W^{m-\kappa_1}}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}(\phi + qW^{r})} = \frac{\widetilde{N}_{st}^{*}\ \tau}{N^{*}\sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}} \exp\left(-\frac{(\mu+\eta-\log W)^2}{2(\sigma^2+\tau^2)}\right) = \hat{\rho}$$

and table 7 then gives us

W	pr	$\hat{\rho}$	$\rho = \hat{\rho}/\hat{\rho}_{pr2}$	W	pr	$\hat{\rho}$	$\rho = \hat{\rho}/\hat{\rho}_{pr2}$
	*pr*1	1.2196E−4	0.9000		*pr*1	9.7690E−4	0.9000
	*pr*2	1.3551E−4	1.0000		*pr*2	1.0855E−3	1.0000
10	*pr*3	1.0841E−4	0.8000	160	*pr*3	8.6836E−4	0.8000
	*pr*4	8.1308E−5	0.6000		*pr*4	6.5127E−4	0.6000
	*pr*5	9.4859E−5	0.7000		*pr*5	7.5981E−4	0.7000
	*pr*6	6.7757E−5	0.5000		*pr*6	5.4272E−4	0.5000
	*pr*1	3.0735E−4	0.9000		*pr*1	3.2046E−3	0.9000
	*pr*2	3.4150E−4	1.0000		*pr*2	3.5608E−3	1.0000
40	*pr*3	2.7320E−4	0.8000	640	*pr*3	2.8485E−3	0.8000
	*pr*4	2.0490E−4	0.6000		*pr*4	2.1365E−3	0.6000
	*pr*5	2.3905E−4	0.7000		*pr*5	2.4925E−3	0.7000
	*pr*6	1.7075E−4	0.5000		*pr*6	1.7804E−3	0.5000

Determination of $\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$, κ_1, $\kappa_0 q/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$, and r:
(vi-*b*) is written as

$$\frac{\kappa_0}{h(\kappa t_l + 1 + \exp(-\kappa t_{00}))}\left(W^{\kappa_1 - m} + \frac{q}{\phi} W^{r+\kappa_1-m}\right) = \frac{1}{stctt} \quad . \tag{xiii}$$

As all parameters for determining ϕ are known now, we find

$\phi_{10} = 469.39 \quad \phi_{40} = 716.76 \quad \phi_{160} = 737.64 \quad \phi_{640} = 720.30$

and (xiii) gives us four nonlinear equations for determining

$\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$, κ_1,
$\kappa_0 q/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$, and r. There are two solutions

$\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00}))$ 49.201. $\kappa_1 - m = -0.8100.$ $\kappa_1 = -0.2500.$
$\kappa_0 q/h/(\kappa t_l + 1 - \exp(-\kappa t_{00})) = 24.605.$ $r + \kappa_1 - m = -0.9601.$ $r = -0.1501.$

and

$\kappa_0/h/(\kappa t_l + 1 - \exp(-\kappa t_{00})) = 52.855.$ $\kappa_1 - m = -0.8640.$ $\kappa_1 = -0.3040.$
$\kappa_0 q/h/(\kappa t_l + 1 - \exp(-\kappa t_{00})) = 18991.$ $r + \kappa_1 - m = -0.8190.$ $r = 0.0450.$

and the first solution is the correct one if r is known to be negative.

Determination of f:
The q which corresponds to our ϕ set is

$q = 24605/49.201 = 500.1$

and the standard f formula gives

$f_{10} = 0.5701 \quad f_{40} = 0.7137 \quad f_{160} = 0.7596 \quad f_{640} = 0.7916.$

As usual all calculations could have been based on *stctr* figures instead of *stctt* figures.

Table 7. Example 4. Stomach data.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$	$stctt$
10	$pr1$	−4.8487	0.50000	6.7431E−1	7.4813E−2
	$pr2$	−3.5658	0.66667	4.3972E−1	
	$pr3$	−2.3875	0.44444	1.8942E−1	
	$pr4$	−0.7839	0.28571	2.8205E−4	
	$pr5$	−4.0564	0.71429	2.7904E−1	
	$pr6$	−2.6790	0.60000	1.1535E−1	
40	$pr1$	−4.1556	0.50000	8.5216E−1	2.8789E−1
	$pr2$	−3.1037	0.66667	8.8124E−1	
	$pr3$	−1.6173	0.44444	4.7892E−1	
	$pr4$	0.2063	0.28571	8.4855E−3	
	$pr5$	−3.6603	0.71429	5.7783E−1	
	$pr6$	−2.1245	0.60000	1.2683E−1	
160	$pr1$	−3.4624	0.50000	5.1961E−1	9.4172E−1
	$pr2$	−2.6416	0.66667	1.1739	
	$pr3$	−0.8471	0.44444	5.2508E−1	
	$pr4$	1.1965	0.28571	8.1597E−2	
	$pr5$	−3.2642	0.71429	8.7152E−1	
	$pr6$	−1.5700	0.60000	8.1541E−2	
640	$pr1$	−2.7693	0.50000	1.2509E−1	3.0166
	$pr2$	−2.1795	0.66667	8.5044E−1	
	$pr3$	−0.0769	0.44444	2.0427E−1	
	$pr4$	2.1867	0.28571	2.0522E−1	
	$pr5$	−2.8681	0.71429	7.8343E−1	
	$pr6$	−1.0155	0.60000	2.5084E−2	

Example 4 is the most general case, and the first examples can be looked upon as special cases. Doing this we get some other ways for the determination of parameters.

Example 1 (p. 5):

The ingestion weight distribution is here

$$\tilde{N}_{st} = \frac{\rho h W^{m-\kappa_1} a N_0 \sqrt{2\pi}\,\sigma}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}(\phi + qW^r)} \frac{1}{\sqrt{2\pi}\,\sigma w} \exp(-(\log w - (-\eta + \log W))^2/2/\sigma^2)$$

$$= \frac{\rho a W^{-1} stctt \sqrt{2\pi}\,\sigma}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)} \frac{1}{\sqrt{2\pi}\,\sigma w} \exp(-(\log w - (-\eta + \log W))^2/2/\sigma^2)$$

a log normal distribution with parameters

$$\hat{\mu} = -\eta + \log W \qquad \hat{\tau}^2 = \sigma^2$$

$$\tilde{N}^*_{st} = \frac{\rho h W^{m-\kappa_1} a N_0 \sqrt{2\pi}\,\sigma}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}(\phi + qW^r)} = \frac{\rho a W^{-1}\, stctt\, \sqrt{2\pi}\,\sigma}{C_0\left(\dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0}\right)}\,.$$

The supplementary data in table 8 now give us the following η, σ^2, and ρ where ρ is found from the formula

$$\rho_{pr} = \frac{\widetilde{N}^*_{st,\,pr}}{a_{pr}\,\sigma_{pr}} \quad \frac{a_{pr2}\,\sigma_{pr2}}{\widetilde{N}^*_{st,\,pr2}} \quad \rho_{pr2}$$

Experiment 1:

pr	η	σ^2	ρ
$pr1$	7.0000	1.0000	0.9000
$pr2$	6.0000	2.0000	1.0000
$pr3$	5.0000	0.8000	0.8000
$pr4$	4.0000	0.4000	0.6000

for W = 10, 40, 160, and 640 g.

Experiment 2:

pr	η	σ^2	ρ
$pr1$	7.0000	1.0000	0.9000
$pr2$	6.0000	2.0000	1.0000

for W = 10, 40, 160, and 640 g.

Example 2 (p. 16):

In this case the ingestion weight distribution is

$$\widetilde{N}_{st} = \frac{\rho\exp(-(\log(W/w) - \eta)^2/2/\sigma^2)N_1 w^{-\nu-1} h W^m}{\dfrac{\kappa}{\kappa t_l + \kappa t_0}\,(\phi + Q)}$$

$$= \frac{\rho N_1 h W^{m-\kappa_1-\nu}\exp\left(\nu\eta + \dfrac{\nu^2\sigma^2}{2}\right)\sqrt{2\pi}\,\sigma}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}\,(\phi + qW^r)}$$

$$* \ \frac{1}{\sqrt{2\pi}\,\sigma\, w}\ \exp\left(-\ \frac{(\log w - (\log W - \eta - \nu\sigma^2))^2}{2\sigma^2}\right)$$

This is also a log normal distribution and the parameters are

$$\hat{\mu} = \log W - \eta - \nu\sigma^2$$
$$\hat{\tau}^2 = \sigma^2$$

$$\widetilde{N}^*_{st} = \frac{\rho N_1 h W^{m-\kappa_1-\nu}\exp(\nu\eta + \dfrac{\nu^2}{2}\,\sigma^2)\sqrt{2\pi}\,\sigma}{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}\,(N_1 C_1 W^{1-\nu} + qW^r)}\ .$$

Table 8 then gives

Environment 1:

$\eta = 5.5000$ and $\sigma^2 = 1.0000$ for all W.

$\widetilde{N}^*_{st}$ gives seven equations of the form

$$\frac{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}}{\rho N_1 h\exp\left(\nu\eta + \dfrac{\nu^2}{2}\,\sigma^2\right)\sqrt{2\pi}\,\sigma}\ (N_1 C_1 W^{1-m+\kappa_1} + qW^{r-m+\kappa_1+\nu}) = \frac{1}{\widetilde{N}^*_{st}}$$

and a test shows that

$$\frac{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0} N_1 C_1}{\rho N_1 h \exp\left(\nu\eta + \dfrac{\nu^2}{2}\sigma^2\right)\sqrt{2\pi}\,\sigma} = 0.23633$$

$$1 - m + \kappa_1 = 0.1901 \qquad \kappa_1 = -0.2499$$

$$\frac{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0} q}{\rho N_1 h \exp\left(\nu\eta + \dfrac{\nu^2}{2}\sigma^2\right)\sqrt{2\pi}\,\sigma} = 3.2558$$

$$r - m + \kappa_1 + \nu = -0.4599 \qquad r = -0.1500$$

fit nicely.

Environment 2:
$\eta = 5.5000$ and $\sigma^2 = 1.0000$ for all W.
$\tilde{N}^*_{st}$ gives a linear regression in $\log W$

$$\log\left(\frac{\kappa_0}{\kappa t_l + \kappa t_0}(N_1 C_1 + q)/(\rho N_1 h \exp(\nu\eta + \frac{\nu^2}{2}\sigma^2)\sqrt{2\pi}\sigma)\right) + (1 - m + \kappa_1)\log W$$
$$= -\log \tilde{N}^*_{st}$$

and the data give

$$\log\left\{\frac{\dfrac{\kappa_0}{\kappa t_l + \kappa t_0}(N_1 C_1 + q)}{\rho N_1 h \exp\left(\nu\eta + \dfrac{\nu^2}{2}\sigma^2\right)\sqrt{2\pi}\,\sigma}\right\} = -1.7327$$

$$1 - m + \kappa_1 = 0.1900 \qquad \kappa_1 = -0.2500.$$

Example 3 (p. 23):
The formulas from example 4 can be used directly and we find using table 8:

First environment:

pr	η	σ^2
$pr1$	7.0000	1.0000
$pr2$	6.0000	2.0000

for W = 10, 40, 160, and 640 g.

W	pr	$\hat{\rho}$	ρ
10	$pr1$	1.9281E−4	0.8999
	$pr2$	2.1425E−4	1.0000
40	$pr1$	5.9988E−4	0.8999
	$pr2$	6.6660E−4	1.0000
160	$pr1$	1.7490E−3	0.9000
	$pr2$	1.9434E−3	1.0000
640	$pr1$	4.7804E−3	0.8999
	$pr2$	5.3120E−3	1.0000

If we rewrite (x) as

$$\frac{\kappa_0}{h\,(\kappa t_l + \kappa t_0)}(N_2 C_2 W^{1-\omega+\kappa_1-m} + q W^{r+\kappa_1-m}) = \frac{\rho N^* \sqrt{\dfrac{\sigma^2 \tau^2}{\sigma^2+\tau^2}}}{\widetilde{N}_{st}^* \exp\left(\dfrac{(\mu+\eta-\log W)^2}{2(\sigma^2+\tau^2)}\right)\tau}$$

we get for each pr four equations for determining $\kappa_0 N_2 C_2/h/(\kappa t_l + \kappa t_0)$, $1-\nu+\kappa_1-m$, $\kappa_0 q/h/(\kappa t_l + \kappa t_0)$, and $r+\kappa_1-m$. Solving the nonlinear equations we get

$pr1$ and $pr2$

$\kappa_0 N_2 C_2/h/(\kappa t_l + \kappa t_0) = 5768.5$
$1-\nu+\kappa_1-m = -0.5875 \qquad \kappa_1 = -0.2497$
$\kappa_0 q/h/(\kappa t_l + \kappa t_0) = 28940$
$r+\kappa_1-m = -0.9596 \qquad r = -0.1499$

Second environment:

pr	η	σ^2
$pr1$	7.0000	1.0000
$pr2$	6.0000	2.0000

for W = 10, 40, 160, and 640 g.

W	pr	$\hat{\rho}$	ρ
10	$pr1$	5.6737E−5	0.9000
	$pr2$	6.3043E−5	1.0000
40	$pr1$	2.1470E−4	0.9000
	$pr2$	2.3856E−4	1.0000
160	$pr1$	8.1246E−4	0.8999
	$pr2$	9.0279E−4	1.0000
640	$pr1$	3.0746E−3	0.9000
	$pr2$	3.4164E−3	1.0000

(x) can be written as

$$\frac{h(\kappa t_l + \kappa t_0)}{\kappa_0(N_2 C_2 + q)} W^{\nu-1-\kappa_1+m} = \frac{\widetilde{N}_{st}^* \exp\left(\dfrac{(\mu+\eta-\log W)^2}{2(\sigma^2+\tau^2)}\right)\tau}{\sqrt{\dfrac{\sigma^2\tau^2}{\sigma^2+\tau^2}}\,\rho N^*}$$

a linear regression in $\log W$ and we get using data for both $pr1$ and $pr2$

$\log(h(\kappa t_l + \kappa t_0)/\kappa_0/(N_2 C_2 + q)) = -11.8822$
$\nu-1-\kappa_1+m = 0.9600 \qquad \kappa_1 = -0.2500$

Table 8. Supplementary stomach data.

Example 1, experiment 1.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	$pr1$	−4.6974	1.0000	3.7565E−1
	$pr2$	−3.6974	2.0000	2.9514E−1
	$pr3$	−2.6974	0.8000	1.1200E−1
	$pr4$	−1.6974	0.4000	4.9496E−3
40	$pr1$	−3.3111	1.0000	6.0931E−1
	$pr2$	−2.3111	2.0000	4.7872E−1
	$pr3$	−1.3111	0.8000	1.8166E−1
	$pr4$	−0.3111	0.4000	8.0283E−3
160	$pr1$	−1.9248	1.0000	6.0458E−1
	$pr2$	−0.9248	2.0000	4.7501E−1
	$pr3$	0.0752	0.8000	1.8025E−1
	$pr4$	1.0752	0.4000	7.9661E−3
640	$pr1$	−0.5385	1.0000	4.9379E−1
	$pr2$	0.4615	2.0000	3.8796E−1
	$pr3$	1.4615	0.8000	1.4722E−1
	$pr4$	2.4615	0.4000	6.5062E−3

Example 1, experiment 2.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	$pr1$	−4.6974	1.0000	8.8777E−2
	$pr2$	−3.6974	2.0000	1.3950
40	$pr1$	−3.3111	1.0000	6.8227E−2
	$pr2$	−2.3111	2.0000	1.0721
160	$pr1$	−1.9248	1.0000	5.2429E−2
	$pr2$	−0.9248	2.0000	8.2384E−1
640	$pr1$	−0.5385	1.0000	4.0289E−2
	$pr2$	0.4615	2.0000	6.3307E−1

Example 2, environment 1.

W	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	−3.6974	1.0000	6.6879E−1
20	−3.0043	1.0000	8.0738E−1
40	−2.3111	1.0000	9.3169E−1
80	−1.6180	1.0000	1.0230
160	−0.9248	1.0000	1.0688
320	−0.2317	1.0000	1.0674
640	0.4615	1.0000	1.0268

Example 2, environment 2.

W	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	−4.3474	1.0000	3.6516
20	−3.6543	1.0000	3.2010
40	−2.9611	1.0000	2.8060
80	−2.2680	1.0000	2.4598
160	−1.5748	1.0000	2.1563
320	−0.8817	1.0000	1.8902
640	−0.1885	1.0000	1.6569

Table 8 continued

Example 3, first environment.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	$pr1$	−4.8487	0.50000	9.9981E−1
	$pr2$	−3.5658	0.66667	6.0622E−1
40	$pr1$	−4.1556	0.50000	1.5599
	$pr2$	−3.1037	0.66667	1.5000
160	$pr1$	−3.4624	0.50000	8.7251E−1
	$pr2$	−2.6416	0.66667	1.8327
640	$pr1$	−2.7693	0.50000	1.7501E−1
	$pr2$	−2.1795	0.66667	1.1063

Example 3, second environment.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	$pr1$	−4.8487	0.50000	2.9420E−1
	$pr2$	−3.5658	0.66667	1.7838E−1
40	$pr1$	−4.1556	0.50000	5.5830E−1
	$pr2$	−3.1037	0.66667	5.3682E−1
160	$pr1$	−3.4624	0.50000	4.0530E−1
	$pr2$	−2.6416	0.66667	8.5136E−1
640	$pr1$	−2.7693	0.50000	1.1256E−1
	$pr2$	−2.1795	0.66667	7.1151E−1

LogW parabolas:

When the environmental weight distribution is log normal it is sometimes possible to determine μ, τ^2, η, and σ^2 from stomach data alone. If we write (x) as

$$N^*_{st} = \frac{\rho N^* h W^m \sqrt{\dfrac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}}}{\dfrac{\kappa_0 W^{\kappa_1}}{\kappa t_l + \kappa t_0} (\phi + q W^r)\tau} \exp\left(-\frac{(\mu + \eta - \log W)^2}{2(\sigma^2 + \tau^2)}\right)$$

$$= \frac{\rho N^* \sqrt{\dfrac{\sigma^2 \tau^2}{\sigma^2 + \tau^2}}\ stctt}{\phi \dfrac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} \tau} \exp\left(-\frac{(\mu + \eta - \log W)^2}{2(\sigma^2 + \tau^2)}\right)$$

we see that

$$\log(\widetilde{N}_{st}^{*}\ W^{1-\nu}/stctt)$$

gives parabolas in $\log W$ for example 3, first and second environment as $\phi = N_2 C_2 W^{1-\nu}$ here. The parabolas give us $(\sigma^2 + \tau^2)$ and $(\mu + \eta)$. Using $\hat{\tau}^2$ and $\hat{\mu}$ we finally get μ, τ^2, η, and σ^2. The calculations are performed in this way:

First environment:

The data give the parabolas:

$pr1$: $\psi(\log W) = 2.7034 + 1.0001 \log W - 0.2500 \log^2 W$

$pr2$: $\psi(\log W) = 2.1450 + 0.8334 \log W - 0.1667 \log^2 W$

giving

$(\sigma^2 + \tau^2) = 2.0000$ and $(\mu + \eta) = 2.0002$ for $pr1$ and
$(\sigma^2 + \tau^2) = 2.9994$ and $(\mu + \eta) = 2.4997$ for $pr2$.

As $\sigma^2 \tau^2 = \hat{\tau}^2(\sigma^2 + \tau^2)$, σ^2 and τ^2 are the solutions of
$x^2 - (\sigma^2 + \tau^2)x + \hat{\tau}^2(\sigma^2 + \tau^2) = 0$.

The solutions are

$\sigma^2 = 1.0000$, $\tau^2 = 1.0000$ for $pr1$
$\sigma^2 = 1.9991$, $\tau^2 = 1.0003$ or $\sigma^2 = 1.0003$, $\tau^2 = 1.9991$ for $pr2$.

μ and η can now be determined by means of (viii). Each W gives us two linear equations:

$$\sigma^2\mu - \tau^2\eta = \hat{\mu}(\sigma^2 + \tau^2) - \tau^2 \log W$$
$$\mu + \eta = (\mu + \eta)$$

The results are

$pr1$: $\eta = 7.0001$ $\mu = -4.9999$. $(W = 10\text{ g})$.
$\eta = 7.0001$ $\mu = -4.9999$. $(W = 40\text{ g})$.
$\eta = 7.0001$ $\mu = -4.9999$. $(W = 160\text{ g})$.
$\eta = 7.0000$ $\mu = -4.9998$. $(W = 640\text{ g})$.

$pr2$: $\eta = 5.9998$ $\mu = -3.5001$ or $\eta = 5.9341$ $\mu = -3.4344$. $(W = 10\text{ g})$.
$\eta = 6.0000$ $\mu = -3.5003$ or $\eta = 6.3960$ $\mu = -3.8963$. $(W = 40\text{ g})$.
$\eta = 6.0002$ $\mu = -3.5005$ or $\eta = 6.8579$ $\mu = -4.3582$. $(W = 160\text{ g})$.
$\eta = 6.0005$ $\mu = -3.5008$ or $\eta = 7.3197$ $\mu = -4.8200$. $(W = 640\text{ g})$.

and the sought parameters are

$(\mu, \tau^2, \eta, \sigma^2) = (-5, 1, 7, 1)$ for $pr1$
$(\mu, \tau^2, \eta, \sigma^2) = (-3.5, 1, 6, 2)$ for $pr2$.

Second environment:

Parabolas:

$pr1$: $\psi(\log W) = -0.2919 + 0.9999\log W - 0.2500\log^2 W$

$pr2$: $\psi(\log W) = -0.8503 + 0.8332\log W - 0.1667\log^2 W$

$(\sigma^2 + \tau^2) = 2.0000$ $(\mu + \eta) = 1.9998$. $(pr1)$. $\sigma^2 = 1.0000$ $\tau^2 = 1.0000$

$(\sigma^2 + \tau^2) = 2.9994$ $(\mu + \eta) = 2.4991$. $(pr2)$. $\sigma^2 = 1.9991$ $\tau^2 = 1.0003$

or $\sigma^2 = 1.0003$ $\tau^2 = 1.9991$

$pr1$: $\eta = 6.9999$ $\mu = -5.0001$. $(W = 10$ g).

$\eta = 6.9999$ $\mu = -5.0001$. $(W = 40$ g).

$\eta = 6.9999$ $\mu = -5.0001$. $(W = 160$ g).

$\eta = 6.9998$ $\mu = -5.0000$. $(W = 640$ g).

$pr2$: $\eta = 5.9994$ $\mu = -3.5003$ or $\eta = 5.9339$ $\mu = -3.4348$. $(W = 10$ g).

$\eta = 5.9996$ $\mu = -3.5005$ or $\eta = 6.3958$ $\mu = -3.8967$. $(W = 40$ g).

$\eta = 5.9998$ $\mu = -3.5007$ or $\eta = 6.8577$ $\mu = -4.3586$. $(W = 160$ g).

$\eta = 6.0001$ $\mu = -3.5010$ or $\eta = 7.3195$ $\mu = -4.8204$. $(W = 640$ g).

$(\mu, \tau^2, \nu, \sigma^2) = (-5, 1, 7, 1)$. $(pr1)$.

$(\mu, \tau^2, \eta, \sigma^2) = (-3.5, 1, 6, 2)$. $(pr2)$.

For the second environment

$$\log(\widetilde{N}^*_{st}\ W^{1-\nu-m+\kappa_1})$$

gives another parabola in $\log W$ for determining $(\sigma^2 + \tau^2)$ and $(\mu + \eta)$:

$pr1$: $\psi(\log W) = -4.4111 + 1.0000\log W - 0.2500\log^2 W$.

$pr2$: $\psi(\log W) = -4.9694 + 0.8333\log W - 0.1667\log^2 W$.

These parabolas give virtually the same results as the former.

Example 5

Table 9 gives data for an environment where $(\mu, \tau^2, \nu, \sigma^2)$ also can be found from a $\log W$ parabola. *Pr* is here feeding on several *pr* species including $pr1$ and $pr2$. $pr1$ and $pr2$ are known to have log normal weight distributions and it is further known that $\phi + qW^r$ is a constant independent of W. (x) now tells us that

$$\log(\widetilde{N}^*_{st}\ W^{-m+\kappa_1})$$

produces a parabola in $\log W$ and the data give parabolas

$pr1$: $\psi(\log W) = -3.2719 + 1.0001\log W - 0.2500\log^2 W$

$pr2$: $\psi(\log W) = -3.8301 + 0.8333\log W - 0.1667\log^2 W$

The parameter set $(\mu, \tau^2, \eta, \sigma^2)$ is thus identical to the values found above for $pr1$ and $pr2$.

Table 9. Example 5. Stomach data.

W	pr	$\hat{\mu}$	$\hat{\tau}^2$	$\tilde{N}^*_{st}$
10	$pr1$	−4.8487	0.50000	6.5086E−1
	$pr2$	−3.5658	0.66667	3.9463E−1
40	$pr1$	−4.1556	0.50000	1.0033
	$pr2$	−3.1037	0.66667	9.6467E−1
160	$pr1$	−3.4624	0.50000	5.9159E−1
	$pr2$	−2.6416	0.66667	1.2427
640	$pr1$	−2.7692	0.50000	1.3345E−1
	$pr2$	−2.1795	0.66667	8.4354E−1

The calculations and determinations in the five examples all behave nicely. This is of course no wonder as the stomach data were constructed from models with all stated assumptions fulfilled. For the sake of completeness all parameter sets used for constructing the data sets, all relevant sets of constants and a summary of the assumptions, data and determinable functions for each example are given here:

Predator *Pr*.

$h = 35 \quad m = 0.56 \quad q = 500 \quad r = -0.15.$

$\kappa_0 = 1519 \quad \kappa_1 = -0.25 \quad \kappa t_l = 0.25 \quad \kappa t_0 = 0.5 \quad \kappa t_{00} = 1.$

$$\kappa t_l + \kappa t_0 = 0.75$$

$$\kappa t_l + 1 + \exp(-\kappa t_0) = 0.64347$$

$$\kappa t_l + 1 + \exp(-\kappa t_{00}) = 0.88212$$

$$\frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 0.85796$$

$$\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 1.17616$$

$$\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + 1 - \exp(-\kappa t_0)} = 1.37088$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_0))} = 67.447$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))} = 49.200$$

$$q\,\frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 428.98$$

$$q\,\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 588.08$$

$$\frac{\kappa_0\, q}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))} = 24600$$

Prey. 11 *pr* species.

*pr*1	$\rho = 0.9$	$\eta = 7$	$\sigma^2 = 1$
*pr*2	$\rho = 1$	$\eta = 6$	$\sigma^2 = 2$
*pr*3	$\rho = 0.8$	$\eta = 5$	$\sigma^2 = 0.8$
*pr*4	$\rho = 0.6$	$\eta = 4$	$\sigma^2 = 0.4$
*pr*5	$\rho = 0.7$	$\eta = 6.5$	$\sigma^2 = 2.5$
*pr*6	$\rho = 0.5$	$\eta = 4.5$	$\sigma^2 = 1.5$
*pr*7	$\rho = 1$	$\eta = 5.5$	$\sigma^2 = 1$
*pr*8	$\rho = 2$	$\eta = 7$	$\sigma^2 = 0.1$
*pr*9	$\rho = 2$	$\eta = 7$	$\sigma^2 = 0.1$
*pr*10	$\rho = 2$	$\eta = 7$	$\sigma^2 = 0.1$
*pr*11	$\rho = 2$	$\eta = 7$	$\sigma^2 = 0.1$

Example 1, 1. experiment:

$N_0 = 1.$

$a_{pr1} = 800 \quad a_{pr2} = 400 \quad a_{pr3} = 300 \quad a_4 = 25$

$$C_0 = 18.208$$

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 15.622$$

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 21.416$$

$$\frac{\kappa_0 q}{C_0 h(\kappa t_l + 1 - \exp(-\kappa t_0))} = 1852.1$$

$$\frac{\kappa_0 q}{C_0 h(\kappa t_l + 1 \exp(-\kappa t_{00}))} = 1351.0$$

$f_{10} = 0.3397 \quad f_{40} = 0.7170 \quad f_{160} = 0.9258 \quad f_{640} = 0.9840$

2. experiment:

$N_0 = 1E6$

$a_{pr1} = 1 \quad a_{pr2} = 10 \quad a_{pr3} 0.1 \quad a_{pr4} = 1 \quad a_{pr5} = 1 \quad a_{pr6} = 0.1$

$$C_0 = 0.28350$$

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 0.24323$$

$$C_0 \frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 0.33344$$

$$C_0 \frac{\kappa_0}{h(\kappa t_l + \kappa t_0)} = 16.405$$

$f_{10} = 0.9999 \quad f_{40} = 1.0000 \quad f_{160} = 1.0000 \quad f_{640} = 1.0000$

Example 2, environment 1:
$N_1 = 200 \quad \nu = 0.5.$

$$C_1 = 0.18158$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_0))C_1 N_1} = 928.61$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))C_1 N_1} = 677.38$$

$$C_1\,\frac{\kappa t_l + 1 - \exp(-\kappa t_0)}{\kappa t_l + \kappa t_0} = 0.15579$$

$$C_1\,\frac{\kappa t_l + 1 - \exp(-\kappa t_{00})}{\kappa t_l + \kappa t_0} = 0.21357$$

$$\frac{q(\kappa t_l + 1 - \exp(-\kappa t_0))}{(\kappa t_l + \kappa t_0)N_1} = 2.1449$$

$$\frac{q(\kappa t_l + 1 - \exp(-\kappa t_{00})}{(\kappa t_l + \kappa t_0)N_1} = 2.9404$$

$f_{10} = 0.2450 \quad f_{20} = 0.3373 \quad f_{40} = 0.4441 \quad f_{80} = 0.5563$
$f_{160} = 0.6630 \quad f_{320} = 0.7553 \quad f_{640} = 0.8289.$

Environment 2:
$N_1 = 200 \quad \nu = 1.15$

$$C_1 = 5.78454$$

$$f = \frac{N_1 C_1}{N_1 C_1 + q} = 0.69823$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_0))h} = 96.597$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))h} = 70.463$$

Example 3, first environment:

pr	μ	τ^2	N^*
*pr*1	−5	1	7502.74
*pr*2	−3.5	1	3488.05
*pr*3	−2	1	140.008
*pr*4	1.5	1	161.161

$N_2 = 1 \quad C_2 = 100$

$\phi_{10} = 166.81 = 100 \ 10^{2/9}$ $\quad f_{10} = 0.3203$
$\phi_{40} = 226.99 = 100 \ 40^{2/9}$ $\quad f_{40} = 0.4412$
$\phi_{160} = 308.89 = 100 \ 160^{2/9}$ $\quad f_{160} = 0.5695$
$\phi_{640} = 420.33 = 100 \ 640^{2/9}$ $\quad f_{640} = 0.6890$

$$\frac{C_2(\kappa t_l + 1 - \exp(-\kappa t_{00}))}{\kappa t_l + \kappa t_0} = 117.62$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))N_2 C_2} = 246.00$$

Second environment:
*pr*1 - *pr*4 as for first environment.

pr	μ	τ^2	N^*
*pr*8	−4.6974	0.1	94187.3
*pr*9	−3.3110	0.1	17019.0
*pr*10	−1.9248	0.1	2879.34
*pr*11	−0.5385	0.1	388.428

$N_2 = 1 \quad C_2 = 2000$

$\phi_{10} = 1415.89 = 2000 \ 10^{-0.15}$ $\quad \phi_{40} = 1150.06 = 2000 \ 40^{-0.15}$
$\phi_{160} = 934.14 = 2000 \ 160^{-0.15}$ $\quad \phi_{640} = 758.76 = 2000 \ 640^{-0.15}$

$$f = \frac{N_2 C_2}{N_2 C_2 + q} = 0.8$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{00}))f} = 61.500$$

Example 4:

pr	μ	τ^2	N^*
*pr*1	−5	1	8000
*pr*2	−3.5	1	4000
*pr*3	−2	1	3000
*pr*4	1.5	1	250
*pr*5	−4	1	3500
*pr*6	−3	1	2500

$\phi_{10} = 469.39$ $\quad f_{10} = 0.5701$
$\phi_{40} = 716.76$ $\quad f_{40} = 0.7137$
$\phi_{160} = 737.64$ $\quad f_{160} = 0.7595$
$\phi_{640} = 720.30$ $\quad f_{640} = 0.7915$

Example 5:
$pr1$-$pr4$ as for example 3, first environment.

pr	μ	τ^2	N^*
$pr8$	−4.6974	0.1	21030.3
$pr9$	−3.3110	0.1	5291.89
$pr10$	−1.9248	0.1	1192.20
$pr11$	−0.5385	0.1	219.614

$\phi_{10} = 446.03 \quad \phi_{10} + q10^r = 800$
$\phi_{40} = 512.48 \quad \phi_{40} + q40^r = 800$
$\phi_{160} = 566.47 \quad \phi_{160} + q160^r = 800$
$\phi_{640} = 610.31 \quad \phi_{640} + q640^r = 800$

Assumptions, data, and determinable parameter functions

Example 1, experiment 1:
Assumptions: $N = aN_0/w$.
Data: Relative abundances (a), $\widetilde{N}_{st}$, *stctr, stctt.*
Determinable quantities:

$\eta, \sigma^2, \rho.$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad \kappa_1 - m$$

$$\frac{\kappa_0 q}{C_0 h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad r + \kappa_1 - 1 - m \quad \text{(i.e. } r\text{)}$$

$$q\frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}, \quad C_0\frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}$$

f

$(\phi = C_0 N_0 W)$

Experiment 2:
Assumptions: $N = aN_0/w, \; f = \underset{\sim}{1}.$
Data: Relative abundances (a), $\widetilde{N}_{st}$, *stctr, stctt.*
Determinable quantities:

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad m - \kappa_1.$$

$\eta, \sigma^2, \rho.$

$$C_0\frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}, \quad C_0\frac{\kappa_0}{\kappa t_l + \kappa t_0}$$

($\phi = C_0 N_0 W$ and very large)

Example 2, environment 1:

Assumptions: One prey species, $N = N_1 w^{-\nu-1}$.
Data: $\tilde{N}_{st}$, *stctr, stctt.*
Determinable quantities:

$$\eta + \sigma^2 \nu,\ \sigma^2.$$

$$C_1 \frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad \kappa_1 - m$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))C_1 N_1}, \quad r - m + \kappa_1 - 1 + \nu$$

$$\frac{q(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}{(\kappa t_l + \kappa t_0)N_1}$$

$$f$$

$$(\phi = C_1 N_1 W^{1-\nu})$$

Environment 2:

Assumptions: One prey species, $N = N_1 w^{-\nu-1}$, $r + \nu = 1$.
Data: N, *stctr, stctt.*
Determinable quantities:

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))f}, \quad m - \kappa_1$$

$$C_1 \frac{\kappa t_l + 1 - \exp(-\kappa t_{o(0)})}{\kappa t_l + \kappa t_0}$$

$$\nu + \sigma^2 \nu,\ \sigma^2$$

$$\frac{\kappa_0}{h(\kappa t_l + \kappa t_0)} \left(C_1 + \frac{q}{N_1} \right)$$

$$(\phi = C_1 N_1 W^{1-\nu})$$

Example 3, first environment:

Assumptions: N log normal (μ, τ^2), $\phi = N_2 C_2 W^{1-\nu}$.
Data: Relative abundances, $\tilde{N}_{st}$, *stctr, stctt.*
Determinable quantities:

$$\nu$$

$$\eta,\ \sigma^2,\ \rho,\ \mu,\ \tau^2$$

$$N_2 C_2 \frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}$$

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad \kappa_1 - m$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))N_2C_2}, \quad r - m + \kappa_1 - 1 + \nu \quad \text{(i.e. } r\text{)}$$

f

Second environment:
Assumptions: N log normal (μ, τ^2), $\phi = N_2C_2W^{1-\nu}$, $\nu + r = 1$.
Data: Relative abundances, $\tilde{N}_{st}$, *stctr, stctt.*
Determinable quantities:

$$\frac{\kappa_0}{fh(\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}, \quad m - \kappa_1$$

ν

$$N_2C_2\,\frac{\kappa t_l + 1 - \exp(-\kappa t_{0(0)})}{\kappa t_l + \kappa t_0}, \quad \frac{\kappa_0(N_2C_2 + q)}{h(\kappa t_l + \kappa t_0)}$$

$\nu, \sigma^2, \rho, \mu, \tau$

Example 4:
Assumptions:N log normal (μ, τ^2).
Data: Relative abundances, (μ, τ^2), $\tilde{N}_{st}$, for all prey species, *stctr, stctt.*
Determinable quantities:

η, σ, ρ, ϕ

$$\frac{\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad \kappa_1 - m$$

$$\frac{q\,\kappa_0}{h(\kappa t_l + 1 - \exp(-\kappa t_{0(0)}))}, \quad r - \kappa_1 - m \quad \text{(i.e. } r\text{)}$$

q

f

Example 5:
Assumptions: N log normal (μ, τ^2), $\phi + qW^r$ = constant.
Data: $\tilde{N}_{st}$.
Determinable quantities:

$\eta, \sigma^2, \mu, \tau^2$

We have now come to an end with the five examples and they show that the assumptions in the model are quite powerful even if they are simple and rather natural. The possibilities are of course not exhausted with the five examples, where the emphasis has been on the determination of the basic preference and digestion parameters. As an example of another use of the model, we can think of the situation where the preference parameters are known. In such cases the formulas evidently deliver determinations of the N^*'s i.e. the relative abundances.

Warnings

As mentioned the determinations behave well in the examples, but the results may be quite different for a set of real data.

In the examples we have used both grouped and ungrouped data, but only for the prey species. Real data will normally have grouped predator weights. The effects of such a grouping have not been investigated as the biases introduced in this way depends in a complicated way on the actual distributions in the *W*-groups. All determinations based on ungrouped data exclusively (i.e. $stctr(t)$ and $\tilde{N}^*_{st}$) are exact except for errors caused by using a limited number of digits in the calculations. These errors are most pronounced for determinations based on nonlinear equations, where the solutions are found by Newton's method. Another problem with nonlinear equations is that there may be more than one solution, and it can be quite tricky to choose the correct solution.

Determinations based on grouped data are generally biased like this:

bias of $\eta = 0$
bias of $\sigma^2 = (\text{group interval})^2/12$

i.e. Sheppard corrections. In example 3 where both stomach and environmental data are grouped both η and σ^2 are biased and rather more than given by Sheppards corrections. The reason for this is that the environmental figures are used as divisors.

A very coarse grouping has been used partly to make the tables less voluminous and partly to give a clear picture of the grouping effects. Strictly the determinations are only valid for data without stochastic components, and when real data are used whether for falsification of the model or parameter estimation one should be very careful when interpreting the results, and a narrow group interval should be used.

The effects of stochastic components have been completely neglected as this effect depends on the nature of the relevant probability distributions, and may be very different for different data sets, and this makes it impossible to give general estimation rules based on the model. It is however very important to make a proper statistical analysis when the stochastic components are present i.e. when the data set is of limited size. The proper estimation may differ considerably from the determination based on the exact model.

Reference

Andersen, K.P. & E. Ursin, 1977: A Multispecies Extension to the Beverton and Holt Theory of Fishing, with Accounts of Phosphorus Circulation and Primary Production. – Meddr Danm. Fisk.- og Havunders., N.S. 7: 319-435.